Spring '01 - Exam 1

(1) (10 pts) Evaluate the limit
$$\lim_{x \to 1} \frac{1 - x}{1 - \sqrt{x}}$$
showing all your steps clearly.

(2) (10 pts) Calculate the derivative $f'(x)$ of the function $f(x) = 1/x^2$ directly from the definition.

(3) (10 pts) You are given the function
$$g(x) = \frac{(1 - 2x)(1 - x)}{(x - 2)(x + 1)}.$$
   (a) Find the equations of all the (horizontal and vertical) asymptotes of the graph.
   (b) Draw the graph of $g(x)$, showing clearly the x-intercepts, y-intercepts and all the asymptotes.

(4) (10 pts) Consider the function
$$h(t) = \begin{cases} 1 + e^t, & t < 0 \\ t + 2, & 0 \le t \le 3 \\ 6 - \frac{3}{t}, & t > 3. \end{cases}$$
   (a) Find all points at which $h(t)$ is discontinuous. Explain.
   (b) Find all points at which $h(t)$ is not differentiable. Explain.
   (c) Draw the graph, clearly labeling these non-smooth points.

(5) (10 pts) Find the equation of the tangent line to the graph of $y = 1 - 2x + 3x^2$ at the point $(1, 2)$.

(6) (10 pts) Let $h(x)$ be differentiable for all x and let $f(x) = (kx + e^x)h(x)$ where k is some constant. Supposing that
$$h(0) = 5, h'(0) = -2, \text{ and } f'(0) = 18,$$
find the value of the constant k.

(7) (10 pts)
   (a) Calculate the derivative of the function $f(x) = xe^x$.
   (b) If $f(x)$ is as above, and
$$g(x) = \frac{f(x)}{x^2 + 1},$$
   find $g'(x)$.