Name (Last, First) _____ **ID #** _____

Signature _____

Lecturer _____ **Section #** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 1** **October 5, 2006**
7:00-8:30 p.m.

Instructions

- **Turn off all cell phones and watch alarms!**
  Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	16	
4	16	
5	16	
6	16	
Free	4	4
TOTAL	100	

1. (16%) The number of *E. coli* bacteria in a Petri dish grows by a constant factor of 16 every hour. At the start of an experiment, there are 1,000 *E. coli* in the dish.

   (a) (4%) How many *E. coli* will there be 2 hours later?

   (b) (8%) Find a formula for the number $Q(t)$ of *E. coli* there will be as a function of the time t, in hours, after the start of the experiment.

   (c) (4%) How long will it take for the number of *E. coli* to reach 1 million? Give your answer to the nearest minute.

2. $(2 \times 8\% = 16\%)$ At time t, in seconds, the coordinate $s(t)$, in feet, of a particle moving on a line is given by

$$s(t) = t^2 - 8t + 18.$$

(a) Find the particle's **average** velocity over each of the following time intervals:

(i) $[4, 4.1]$ Calculate your answer as a single number.

(ii) $[4, 4 + \Delta t]$ where $\Delta t > 0$. Simplify your answer.

(b) Find the particle's instantaneous velocity at $t = 4$. Use the meaning of velocity in terms of limits.

3. (16%) Let $f(x) = \dfrac{x+2}{\sqrt{9\,x^2-1}}$.

(a) (4%)What is the domain of f?

(b) (6%) By evaluating relevant limits, find an equation of each vertical asymptote of the graph of f. (If there are none, say so!)

(c) (6%) By evaluating relevant limits, find an equation of each horizontal asymptote of the graph of f. (If there are none, say so!)

4. $(2 \times 8\% = 16\%)$ The functions f and g are defined by:

$$f(x) = \begin{cases} x^2 + 3x & \text{if } x \neq 1, \\ 0 & \text{if } x = 1, \end{cases} \qquad g(x) = \begin{cases} 5 - 6x & \text{if } x \neq 1, \\ 3 & \text{if } x = 1. \end{cases}$$

(a) Are f and g continuous at $x = 1$? Why or why not?

(b) Is $f + g$ continuous at $x = 1$? Why or why not?

5. (16%) *The parts of this question are* not *related.*

(a) (10%) Use Limit Laws to determine:

$$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2}$$

(b) (6%) Let $f(x) = 5\,x + 4$. so that, of course,

$$L = \lim_{x \to 2} f(x) = 14.$$

For $\epsilon = 0.01$, find a corresponding value of δ such that, for all $x \neq 2$:

$$\text{if } 2 - \delta < x < 2 + \delta \text{ , then } L - \epsilon < f(x) < L + \epsilon$$

Do this *algebraically* and *without* graphing the function.

6. $(2 \times 8\% = 16\%)$

(a) Use the ***definition*** of derivative to find the derivative of $f(x) = \dfrac{x-1}{x}$.

(b) Find an equation of the tangent line to the graph of $y = \dfrac{x-1}{x}$ at the point where $x = 2$. [In case you were unable to do (a), you may use the fact that $f'(2) = 1/4$.]

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