NAME ______________________ ID # __________

PROFESSOR ______________________ SECTION # ______

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Exam 1** **October 13, 2004**
6:30–8:00 p. m.

Instructions

- There are seven (7) questions.
- Do all work in this exam booklet. If your work won't fit in the space provided, clearly indicate where it is continued. (You may use backs of pages and the blank page attached at the end.)
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- Do *not* write anything in the table below.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	14	
2	15	
3	14	
4	14	
5	15	
6	14	
7	14	
TOTAL	100	

1. Let

$$f(x) = \frac{3\,x - 1}{x + 2}.$$

   (a) (3%) What is the domain of f?

   (b) (8%) Find a formula for $f^{-1}(x)$, where f^{-1} is the inverse of f.

   (c) (3%) What is the range of that inverse function?

2. ($3 \times 5\% = 15\%$) Find each of the following limits; if the limit does not exist, say so and tell why. Do this *without* using your calculator. (You may want to use your calculator to confirm your answers).

   (a) $\displaystyle\lim_{x \to 0} \frac{x-3}{x^2-9}$

   (b) $\displaystyle\lim_{x \to 3} \frac{x-3}{x^2-9}$

   (c) $\displaystyle\lim_{x \to 0} \frac{\sqrt{9+x}-3}{x}$

3. ($2 \times 7\% = 14\%$) The temperature of a freshly poured cup of coffee was measured at two-minute intervals. Here is some of the data collected for the coffee's temperature T, measured in degrees Celsius, as a function of the elapsed time t, in minutes, after the coffee was poured.

t	0	2	4	6	**8**	10	12	14	16
T	93.99	84.18	76.23	68.60	**62.21**	56.64	51.96	48.08	44.56

(a) Using only this data, find the *best* estimate you can for the **rate**, in degrees Celsius per minute, at which the coffee's temperature was changing at the time $t = 8$ minutes. Indicate how you obtain your estimate. Do *not* round it. (Do *not* try to fit a formula to this data!)

(b) Actually, the rate at time $t = 8$ at which T was changing with respect to t was exactly -3.15 °C. Write an equation for the tangent line to the graph of the function T of t at the point where $t = 8$. (The variables are time t and temperature T.)

4. Let $f(x) = \dfrac{x^3 + x}{x^2 - 4}$.

(a) (6%) Is f even, odd, or neither? Why? Do this *without* using your calculator. (You may want to use your calculator to confirm your answer).

(b) (8%) What lines, if any, are horizontal and vertical asymptotes of the graph of f? Justify your answer. Do this *without* using your calculator. (You may want to use your calculator to confirm your answer).

5. Let $f(x) = x^2 + 3\,x + 5$.

(a) (5%) Use the **definition** of the derivative to express the derivative $f\,'(2)$ of this function f at 2 as a limit (but do *not* yet evaluate that limit):

$f\,'(2) \;\; =$ ____________________

(b) (10%) Now evaluate the limit you wrote in (a) so as to determine the derivative $f\,'(2)$. Do this *without* using your calculator. (You may want to use your calculator to confirm your answer).

6. *The two parts of this question are* not *directly related.*

(a) (8%) What constant c, if any, makes the function

$$f(x) = \begin{cases} e^{c\,x} & \text{if } x < 1, \\ 2\,x + 3 & \text{if } x \geq 1. \end{cases}$$

continuous at $x = 1$? Why? (Sketches of graphs will not suffice. Do *not* use trial-and-error!)

(b) (6%) Use the Intermediate Value Theorem to explain why the equation

$$\sqrt{x^3 + 2} - x = 1$$

has some solution between $x = -1$ and $x = 1$. (Do *not* attempt to solve the equation. Sketches of graphs will *not* serve as an explanation.)

7. The radioactive compound UConn-131 decays with time, losing half its mass every 567 years. Suppose you have 3 grams of UConn-131 now.

   (a) (7%) Find a formula for the mass M, in grams, of UConn-131 as a function of time t, in years from now.

   (b) (7%) When will there be only one gram remaining? Give your answer to the nearest year.

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