1. (a) [8%] $$\text{avg } v = \frac{h(5.2)-h(5)}{5.2-5} = \frac{[1200-4.9\,(5.2)^2]-[1200-4.9\,(5)^2]}{0.2} = \frac{-4.9[(5.2)^2+(5)^2]}{0.2} = -4.9\frac{2.04}{0.2} = \boxed{-49.98 \text{ m/s}}.$$

   (b) [8%] $$\text{avg } v = \frac{h(5+\Delta t)-h(5)}{\Delta t} = \boxed{\frac{[1200-4.9\,(5+\Delta t)^2]-[1200-4.9\,(5)^2]}{\Delta t} \text{ (m/sec)}}$$

   The question did not ask you to simplify the result, but if you do so, you should obtain:

   $$= \frac{-4.9\left(25+10\Delta t+(\Delta t)^2\right)+4.9(25)}{\Delta t} \qquad = \frac{-4.9(10\Delta t+(\Delta t)^2}{\Delta t}$$
   $$= \frac{-4.9\,\Delta t(10+\Delta t)}{\Delta t} = -4.9\,(10+\Delta t) \text{ (m/sec)}$$

2. (a) [10%] The desired slope m is given by:

   $$\begin{aligned} m &= \lim_{h\to 0} \frac{f(4+h)-f(4)}{h} \\ &= \lim_{h\to 0} \frac{[(4+h)^2-3(4+h)]-[4^2-3(4)]}{h} \\ &= \lim_{h\to 0} \frac{16+8h+h^2-12-3h-16+12}{h} \\ &= \lim_{h\to 0} \frac{5h+h^2}{h} \\ &= \lim_{h\to 0} \frac{h(5+h)}{h} \\ &= \lim_{h\to 0} (5+h) \\ &= 5+0 = \boxed{5}. \end{aligned}$$

   (b) [6%] The form of the tangent line is

   $$y - f(4) = m(x-4).$$

   The tangent line has slope $m = 5$ and goes through the point

   $$\left(4, f(4)\right) = \left(4, 4^2-3(4)\right) = (4,4).$$

   So its equation is

   $$\boxed{y-4 = 5(x-4),}$$

   That form is OK. So is a simplified form such as: $y = 5\,x - 16$

3. (a) [8%] Factor the denominator to obtain

$$f(x) = \frac{4x^2+5}{(x-1)\,(x-5)}.$$

Then f is continuous at every $x \neq 1, 5$, so the only possible vertical asymptotes are at $x = 1$ and $x = 5$.

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{4x^2+5}{(x-1)\,(x-5)} = -\infty$$

because, for $x > 1$ and $x \approx 1$, we have $4x^2 + 5 \approx 4(1)^2 + 5 = 9 > 0$, while $x - 1 > 0$ and $x - 5 < 0$—so that $(x-1)(x-5) > 0$—with $x - 1 \approx 0$.

[You could similarly see that $\lim\limits_{x \to 1^-} f(x) = +\infty$. But finding that *one* of the two one-sided limits is infinite is enough to establish that there is an asymptote at $x = 1$.]

Next,

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} \frac{4x^2+5}{(x-1)\,(x-5)} = +\infty$$

because, for $x > 5$ and $x \approx 5$, we have $4x^2 + 5 \approx 4(5)^2 + 5 = 105 > 0$, while $x - 1 > 0$ and $x - 5 > 0$—so that $(x-1)(x-5) > 0$—with $x - 5 \approx 0$.

[You could similarly see that $\lim\limits_{x \to 5^-} f(x) = -\infty$. But again it's enough to find *one* of the two one-sided limits.]

Hence **the lines $x = 1$ and $x = 5$ are the vertical asymptotes**.

(b) [8%]

$$\begin{aligned}\lim_{x \to \infty} f(x) &= \lim_{x \to \infty} \frac{4x^2+5}{x^2-6x+5} \\ &= \lim_{x \to \infty} \frac{4+5/x^2}{1-6/x+5/x^2} \\ &= \frac{4+0}{1-0+0} = 4\end{aligned}$$

Similarly,

$$\lim_{x \to -\infty} f(x) = 4.$$

Hence **the line $y = 4$ is the one and only horizontal asymptote**.

(Here you *did* need to determine the limit in the other direction, as $x \to -\infty$, too.)

4. (a) [8%]

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{x-3}$$
$$= \lim_{x\to 3} (x+3) = 3+3 = \boxed{6} \qquad \text{(by Direct Substitution))}$$

(b) [8%]

$$\lim_{x\to 4} \frac{2-\sqrt{x}}{x^2(4-x)} = \lim_{x\to 4} \frac{2-\sqrt{x}}{x^2(4-x)} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} \qquad \text{(rationalizing numerator)}$$
$$= \lim_{x\to 4} \frac{2^2-(\sqrt{x})^2}{x^2(4-x)} = \lim_{x\to 4} \frac{4-x}{x^2(4-x)(2+\sqrt{x})}$$
$$= \lim_{x\to 4} \frac{1}{x^2(2+\sqrt{x})}$$
$$= \frac{1}{4^2(2+\sqrt{4})} = \boxed{\frac{1}{64}} \qquad \text{(by Direct Substitution)}$$

(by the product rule and continuity of cos).

5. (a) [6%] For every $\epsilon > 0$, there exists some δ such that:

if $\boxed{x \neq 1 \text{ and } 1-\delta < x < 1+\delta}$, then $\boxed{4-\epsilon < f(x) < 4+\epsilon}$.

You could also complete the statement in the form:

if $\boxed{0 < |x-1| < \delta}$, then $\boxed{|f(x)-4| < \epsilon}$.

In either of those forms, you could substitute the actual expression $(6x-2)$ for $f(x)$.

(b) [8%] You may do this with or without absolute values. Here's the way without them. We want to make $4-\epsilon < f(x) < 4+\epsilon$, that is,

$$4-0.1 < 6x-2 < 4+0.1 \qquad (1)$$

when x is sufficiently close to 1 (but $x \neq 1$). Now condition (**??**) is equivalent in turn to each of:

$$6-0.1 < 6x < 6+0.1$$
$$1-\frac{0.1}{6} < x < 1+\frac{0.1}{6}$$

So we take

$$\boxed{\delta = \frac{0.1}{6}}.$$

or any smaller positive number. Of course $0.1/6 \approx 0.01666667$. However, if you want to give a single decimal as the answer, it would be *wrong* to take $\delta = 0.01666667$ or $\delta = 0.0166667$, etc. Instead, you would have to use for δ a value that is definitely *less than* $0.1/6$. For example, it would be OK to take $\delta = 0.166666$.

6. (a) [8%] That f is continuous at $x=1$ means $\lim_{x\to 1} f(x)$ exists and $\lim_{x\to 1} f(x) = f(1)$.
The one-sided limits are:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (kx+4) = k(1)+4 = k+4,$$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x^2 = 2(1)^2 = 2.$$

For $\lim_{x\to 1} f(x)$ to exist we need these two one-sided limits to be equal:

$$k+4=2,$$

that is,

$$k=-2.$$

Thus for $k=-2$ we have $\lim_{x\to 1} f(x)$ exists, and $\lim_{x\to 1} f(x) = 2$. Since $f(1)=2$, then

$$\lim_{x\to 1} f(x) = f(1).$$

Hence f is continuous at $x=1$ exactly in the case that $\boxed{k=-2}$.

(b) [8%] We now know

$$f(x) = \begin{cases} -2x+4 & \text{if } x<1, \\ 2x^2 & \text{if } 1 \le x, \end{cases}$$

The question is whether $\lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1}$ exists.
The one-sided limits of the difference quotients are:

$$\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{(-2x+4)-2}{x-1} = \lim_{x\to 1^-} \frac{-2x+2}{x-2} = \lim_{x\to 1^{-1}} \frac{-2\,(x-1)}{x-1}$$
$$= \lim_{x\to 1^{-1}} (-2) = -2$$

and

$$\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{2x^2-2}{x-1} = \lim_{x\to 1^+} \frac{2(x^2-1)}{x-1} = \lim_{x\to 1^+} \frac{2(x+1)(x-1)}{x-1}$$
$$= \lim_{x\to 1^+} 2(x+1) = 2(1+1) = 4.$$

$\left[\text{You could have worked instead with one-sided limits of } \dfrac{f(1+h)-f(1)}{h}.\right]$

Since these two one-sided limits are *not* the same, then $\boxed{f \textbf{ is } \textbf{\textit{not}} \textbf{ differentiable at } x=1}$.

Note: To say simply that f is not differentiable at $x=1$ because its graph has a corner there is *not* enough—and will receive only very partial credit—because the question remains as to how you know for sure that there's a corner there. (And to determine that, you have to calculate the one-sided limits of the difference quotient, as above.)