DEPARTMENT OF MATHEMATICS AND STATISTICS UNIVERSITY OF MASSACHUSETTS

NAME (print): _____

SIGNATURE: _____

8 DIGIT STUDENT #: ____ ____ ____ ____ ____ ____ ____ ____

CIRCLE the name of your instructor below: Rudvalis Sect. 1 MWF 9:05

Rudvalis Sect. 2 MWF 10:10, Kevrekidis Sect. 3 TuTh 1:00

DIRECTIONS:

- **This is a 120 minute exam. It consists of 25 multiple choice questions. Your (percentage) score on this exam is 4 times the number of correct responses. Your responses must be recorded on the green "bubble" sheets using a No. 2 pencil.**

- **On your green "bubble" sheet you must write your name at the top left side in the section labeled NAME and your 8 digit student number in the section labeled IDENTIFICATION NUMBER in the middle of the bottom left using the spaces labeled A through H. In addition you must write in the SPECIAL CODE for this exam (200504) in the section with that label. You must also fill in the "bubbles" for each of the above and also "bubble" in your section number (1, 2, 3 from above) in the column labeled GRADE or EDUC. Do not write or "bubble" in the sections for SEX or BIRTH DATE. Your responses to each of the 25 questions must be made by filling in the appropriate bubble on your answer sheet. The grading machine reads only the bubbles you have filled out in GRADING your exam so entering these bubbles correctly is vital to correctly recording your performance. All bubbles must be filled in solidly using a #2 pencil.**

- **You are allowed to use any kind of calculator for this exam. You are responsible for having a working calculator and knowing how to use it and also for having at least one No. 2 pencil. YOU MAY NOT SHARE A CALCULATOR WITH ANOTHER STUDENT DURING THIS EXAM. You may also use a two-sided 8.5 x 11 page as a review sheet during the exam. If you need more paper for work than is provided on the exam page, raise your hand and we will supply you with scratch paper. You may not have anything else on your desk except your STUDENT ID which may be checked when you hand in your exam.**

- **DO NOT LEAVE YOUR SEAT once you have started the exam until you are ready to turn it in. If you have a question or need extra paper raise your hand and we will come to you.**

- **When you have FINISHED, turn in the green "bubble" sheet AND this COVER PAGE of the test booklet. Take the test booklet with you and mark your answers in the test booklet before you turn in the exam so that you can figure out your score as the correct answers will be put online after the exam. Grades will NOT be posted and do not call the department or your instructor for grades as no grades will be given over the phone. BEST OF LUCK!**

1. The DERIVATIVE of $y = f(x) = (x^2 + 4) \ln x$ is:

a) $2x \ln x$
b) $(x^2 + 4)(1/x)$
c) $2x \ln x + (x^2 + 4)/x$
d) $2x \ln x + x^2 + x/4$
e) $2x$

2. The DERIVATIVE of $y = f(x) = e^{2x} / (x^3 + 6)$ is:

a) e^{2x} / x^2
b) $e^{2x} (2x^3 + 3x^2 + 12) / (x^3 + 6)^2$
c) $e^{2x} (2x^3 - 3x^2 + 12) / (x^2 + 6)$
d) $2e^{2x} / (3x^2 + 6)$
e) $e^{2x} (2x^3 - 3x^2 + 12) / (x^3 + 6)^2$

3. The value of the integral $\int_0^3 3x/(x^2 + 2)\, dx$ is approximately:

a) 0.853
b) 1.704
c) 2.557
d) 3.409
e) 4.263

4. A population function P(t) has derivative $P'(t) = \ln (t^2 + 1)$ where t is measured in years and P(t) is measured in millions of people. In particular, P(t) is increasing for all $t \geq 0$. Given all this information, the total (i.e. net) change in population between years $t = 0$ and $t = 5$ years is:

a) 3.258 millions /year
b) 3.258 millions
c) 9.037 millions
d) 9.037 millions/year
e) 0.711 millions

5. Find the area of the finite plane region bounded by (i.e. between) the curves $y = f(x) = 9 - x^2$ and $y = g(x) = x^2 - 9$.

a) 18 b) 36 c) 72 d) 144 e) None of the above is correct.

6. Given the MARGINAL Cost is $C'(t) = 0.5\,q^2 + 2q + 2$ in units \$/item, find the change in cost when production is increased from $q = 6$ to $q = 7$ items.

a) 3.2 \$
b) 36.2 \$
c) 55.8 \$
d) 84.0 \$
e) 120.2 \$

7. Suppose a bottle factory has fixed cost 1000 \$ and marginal cost $C'(q) = 12q^2$. Using this information compute the TOTAL cost of production when $q = 10$ bottles are produced.

a 1000 \$
b) 2000 \$
c) 3000 \$
d) 4000 \$
e) 5000 \$

8. Suppose the velocity of a car is given by $v(t) = 20\,t(4 - t)$ miles/hr. Find the TOTAL distance traveled by this car between time $t = 0$ and $t = 4$. HINT: $v(t)$ is <u>non-negative</u> for all t between $t = 0$ and $t = 4$.

a) 0 miles
b) 213.33 miles / hr
c) 213.33 miles
d) -213.33 miles / hr
e) -213.33 miles

9. Consider the function y = f(x) whose graph is given at the right:

Insert Panos graph here

Which of the numbers below BEST approximates $\int_{10}^{20} f(x)\,dx$:

a) 15
b) 45
c) 60
d) 85
e) 100

10. Suppose the graph of y = f(x) is given at the right:
At which of the marked points is the derivative $y' = f'(x)$ the smallest (i.e. least)

Replace Panos graph with the one from **#11** *from Fall 2002*

a) B b) E c) F d) G
e) The answer cannot be determined without knowing f(x).

11. Given $f(x) = \ln(3x)$ and $g(x) = 2x^2 + 1$, the value of $g(f(2))$ is closest to:

a) 1.792 b) 3.296 c) 4.332 d) 7.421 e) 9

12. Suppose a bacterial culture has continuous exponential growth with growth rate k = 0.2 where time t is measured in hours. If possible, determine how long it will take for the number of bacteria to be 10 times the initial population.

a) 7.7 hours b) 11.5 hours c) 22.6 hours d) 30 hours
e) Impossible to determine

13. For the function $y = f(x) = x^3 - (9/2)x^2 + 6x + 1$ defined on the interval $0 \le x \le 3$ which of the following statements is TRUE:

a) The global minimum is at x = 0 and y = 2.
b) The global maximum is at x = 1 and y = 3.5.
c) The global minimum is at x = 2 and y = 3.
d) The global maximum is at x = 3 and y = 5.5.

e) The global maximum is at x = 3 and y = 5.

14. The function $C(t) = 4t\,e^{-2t}$ has a POINT OF INFLECTION when:

a) t = 1 b) t = 2 c) t = 3 d) C = 0.406 e) C = 0.11

15. Suppose you win the lottery and your winnings are paid out as follows: You receive 1,000,000 $ one year from today and an additional 1,000,000 $ two years from today. Assume the annual interest rate is 4% for investments COMPOUNDED ANNUALLY during these two years. Given this information and rounding off to the nearest 1000 $, what is the present value (i.e. current value) of these future payments?

a) 2,000,000 $ b) 1,923,000 $ c) 1,886,000 $ d) 1,849,000 $ e) 1,805,000 $

16. Suppose the concentration C(t) of a drug is given by $30\,t\,e^{-t/5}$ where time t is measured in hours and C is measured in μgrams (i.e. micrograms) / cc (i.e. cubic centimeter). At time t = 4 hours C(t) is:

a) increasing b) decreasing c) a local minimum d) a global maximum
e) none of the previous answers is correct.

17. Suppose money has been invested in an account where the annual interest rate is r and where the value of your account is compounded continuously. If it takes 17 years for the value of your investment to double, then the value of r is closest to:

a) .03 b) .035 c) .04 d) .045 e) .05

18. The total cost C(q) for producing q units of a certain product is given by the graph at the right. For this cost

Insert graph from #13

function C(t), the marginal cost C′(30) at production level q = 30 units is closest to: *Fall 2002 here.*

a) 25 $/unit b) 45 $/unit c) 100 $/unit

d) 2200 $ e) 54,000 $

19. For the total cost function C(t) in the graph above, the slope of the secant line connecting the points at q = 20 and q = 40 is closest to which of the possibilities below:
HINT: The slope of the secant line is the average rate of change of the function.

a) 20 $/unit b) 50 $/unit c) 150 $/unit d) 3000$ e) 5000 $

20. For the function $y = f(x) = x^{1/3}$ the derivative is:

a) $(1/3)\,x$ b) $(1/3)\,x^{1/3}$ c) $(1/3)\,x^{-2/3}$ d) $x^{1/3} \ln x$ e) $x^{1/3} \ln (1/3)$

21. The equation of the tangent line $y = g(x)$ to the graph of $y = x^{1/3}$ at the point (8,2) is:

a) $y = g(x) = 8 + (1/12)(x - 2) = 8 + (.0833)(x - 2)$
b) $y = g(x) = 2 + (2 \ln 8)(x - 8) = 2 + (4.16)(x - 8)$

c) $y = g(x) = 2 + (2/3)(x - 8) = 2 + (.6667)(x - 8)$

d) $y = g(x) = 8 + (2/3)(x - 2) = 8 + (.6667)(x - 2)$

e) $y = g(x) = 2 + (1/12)(x - 8) = 2 + (.0833)(x - 8)$

22. [This problem is a continuation of numbers 20 and 21 from above.] Use the tangent line $y = g(x)$ found above to compute the local linear APPROXIMATION $g(9)$ for $f(9) = 9^{1/3}$. REMARK: Do not compute $9^{1/3}$ by your calculator as this problem requires you to find the local linear approximation rather than the exact value of $9^{1/3}$. This approximation method gives a value closest to:

a) 2.1333 b) 2.0833 c) 2.080 d) 2.0667 e) 2

23. Suppose $y = g(t)$ is an unspecified function which we know is increasing on some interval (a, b) and we are also given that the derivative $g'(t)$ is decreasing on this interval. Which of the following statements is TRUE:

a) g(t) has no points of inflection in the interval (a,b).
b) g(t) has at least one local maximum somewhere in the interval (a,b).
c) g(t) has at least one local minimum somewhere in the interval (a,b).
d) g(t) has at least one point of inflection somewhere in the interval (a,b).
e) All of the previous statements are false.

24. A deer suddenly appears on a road 300 feet ahead, and seems paralyzed by your headlights. You hit the brakes immediately, and your velocity decreases as in the table below until you come to a complete stop 6 seconds later.

t *(sec)*	**0**	**2**	**4**	**6**
v(t) *(ft/sec)*	**80**	**50**	**15**	**0**

The following statements specify whether (Yes) or not (No) you hit the deer and then give an explanation. You must determine which of the statements below is true:

a) Yes, because a lower bound (i.e. underestimate) for your stopping distance is more than 300 ft.
b) No, because a lower bound (i.e underestimate) for your stopping distance is less than 300 ft.
c) No, because an upper bound (i.e. overestimate) for your stopping distance is less than 300 ft.
d) Yes, because an upper bound (i.e. overestimate) for your stopping distance is more than 300 ft.
e) Can't tell for sure because a lower bound is below 300 ft and an upper bound is above 300 ft.

HINTS: (1) You will want to sketch an approximate graph of velocity versus time during the six seconds and estimate the stopping distance (i.e. the area under this graph) in order to interpret the possible responses above.
(2) Whether or not you hit the deer you can ignore any deceleration of your vehicle that might be caused by a possible impact with the deer.

25. The graph of the function y = f(x) on the interval [0, 100] appears at the right. Furthermore, you are also given that the AREAS of the regions labeled A, B and C respectively are 25, 50 and 10.

Insert the graph from #22 Fall 2002 Final here

Given this information, the value of $\int_0^{100} f(x)\,dx$ **is:**

a) 15
b) 25
c) 50
d) 85
e) 500