

1. (30 points) Compute the following (in cartesian or polar form):

a) ^{8 pts} The polar form of $z := \frac{2}{-\sqrt{3} + \sqrt{3}i} = \frac{2(-\sqrt{3} - \sqrt{3}i)}{(\sqrt{3})^2 + (\sqrt{3})^2} = -\frac{1}{\sqrt{3}}(1+i) = -\frac{1}{\sqrt{3}}\sqrt{2}e^{i\pi/4}$

$$= -\frac{\sqrt{2}}{\sqrt{3}}e^{i\pi/4} = \frac{\sqrt{2}}{\sqrt{3}}e^{5\pi i/4}$$

^{4 pts}

b) $|z^6|$, where z is given in part a.

$$\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^6 = \left(\frac{2}{3}\right)^3 = \frac{8}{27} \quad (-\pi, \pi]$$

^{6 pts}

c) $\text{Log}(a^6)$, where $a = 2e^{-i[2\pi/11]}$.

$$a^6 = 2^6 e^{-i \cdot 12\pi/11} = 2^6 e^{i(10\pi/11)}$$

$$\text{Log}(a^6) = \underbrace{\ln(2^6)}_{6 \ln(2)} + i \frac{10\pi}{11}$$

^{6 - pts}
 d) Find all values of $(1-i)^{1/6}$. How many different values are there?

$$(1-i)^{1/6} = e^{\frac{1}{6} \log(1-i)} = e^{\frac{1}{6} [\ln(\sqrt{2}) + i(-\frac{\pi}{4} + 2k\pi)]}, \quad k \text{ integer,}$$

$$= \left(\frac{1}{2}\right)^{1/6} \cdot e^{-\frac{\pi i}{24}} \cdot e^{i k \frac{\pi}{3}}, \quad 0 \leq k \leq 5$$

$$= 2^{1/12} e^{-\frac{\pi i}{24}} \cdot e^{i k \frac{\pi}{3}}, \quad \text{" There are 6 different values.}$$

^{6 - pts}
 e) Find all values of 2^{3-i} . How many different values are there? Justify!

$$2^{3-i} = e^{(3-i) \log(2)} = e^{(3-i) [\ln(2) + 2k\pi i]}, \quad k \text{ integer,}$$

$$= e^{(3 \ln(2) + 2k\pi) + i(-\ln(2) + 6k\pi)}, \quad k \text{ integer,}$$

$$= e^{[3 \ln(2) + 2k\pi]} \cdot e^{-\ln(2)i}, \quad \text{" "}$$

There are ∞ -many values

