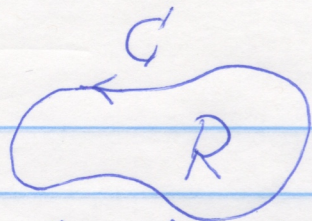


Sec 49 page 160 #7:



C is a positively oriented simple closed contour.
 R = region enclosed by C (= Domain enclosed by C and points on C). We show that

$$\text{Area}(R) = \frac{1}{2i} \int_C \bar{z} dz$$

$$\frac{1}{2i} \int_C \bar{z} dz = \frac{1}{2i} \int_C \underbrace{(x-iy)}_{x+iy} [\int_C x dx + y dy + i \int_C -y dx + x dy] =$$

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$$= \frac{1}{2i} \left[\iint_R \underbrace{\frac{d}{dx}(y) - \frac{d}{dy}(x)}_0 dx dy + i \iint_R \underbrace{\frac{d}{dx}(-y) - \frac{d}{dy}(x)}_2 dx dy \right]$$

$$= \frac{1}{2i} i \iint_R 2 dx dy = \text{Area}(R).$$