Additional Problem on Cauchy-Goursat's Theorem for simply connected domains

Let D be a simply connected domain and f a function analytic on D. Assume that $f(z) \neq 0$, for all z in D.

a) Show that there exists a branch of $\log(f(z))$ defined on D. I.e., show that there exists a function $G(z)$ analytic in D, such that

$$e^{G(z)} = f(z), \tag{1}$$

for all z in D.

Hint: Show first that the function $g(z) := \frac{f'(z)}{f(z)}$ has an antiderivative $\widetilde{G}(z)$ in D. Choose z_0 in D and a complex number c satisfying $e^c = f(z_0)$ and normalize

$$G(z) := \widetilde{G}(z) + c - \widetilde{G}(z_0)$$

so that G is another antiderivative of g and $G(z_0) = c$. Then show that the above displayed Equation (1) holds for all z in D. Do this by proving that $e^{-G(z)} f(z)$ is the constant function on D with value 1. Note that in class we did the special case where $D = \mathbb{C} \setminus \{te^{it} \ : \ t \in \mathbb{R}, t \geq 0\}$, $z_0 = 1$, and $f(z) = z$ to conclude the existence of a branch of $\log(z)$ on D. Argue similarly.

b) Show that there is a branch of $\sqrt{f(z)}$ in D. I.e., there exists a function $h(z)$, analytic on D, such that $h^2(z) = f(z)$, for all $z \in D$.