$$\cosh(y) = \frac{e^y + e^{-y}}{2} \quad \sinh(y) = \frac{e^y - e^{-y}}{2}$$

<u>Sec 23 page 71 problem 2 (d)</u>

$$f(z) = \underbrace{\cos(x)\cosh(y)}_{u(x,y)} + i\underbrace{(-\sin(x)\sinh(y))}_{v(x,y)}$$

The partials of u and v (exist and) are continuous everywhere:

$$u_x = -\sin(x)\cosh(y)$$

$$\left.\begin{aligned} u_y &= \cos(x)\sinh(y) \\ v_x &= -\cos(x)\sinh(y) \end{aligned}\right\} \boxed{\text{C.R. 2}} \; u_y = -v_x$$

$$v_y = -\sin(x)\cosh(y) \underset{\boxed{\text{C.R. 1}}}{=} u_x$$

Hence, the Cauchy-Rieman Eq are satisfied and $f'(z)$ exists and the Theorem in Sec 22 yields

$$f'(x+iy) = u_x(x,y) + i\,v_x(x,y) =$$

$$= \underbrace{-\sin(x)\cosh(y)}_{\tilde{u}(x,y)} + i\underbrace{(-\cos(x)\sinh(y))}_{\tilde{v}(x,y)}$$

$$\tilde{u}_x = -\cos(x)\cosh(y) \underset{\boxed{\text{C.R. 1}}}{=} \tilde{v}_y$$

$$\tilde{u}_y = -\sin(x)\sinh(y)$$

$$\tilde{v}_x = \sin(x)\sinh(y) \underset{\boxed{\text{C.R. 2}}}{=} -\tilde{u}_x.$$

Hence, the partials of $\tilde{u}$ and $\tilde{v}$ are continuous everywhere and satisfy the Cauchy-Riemann Equations. It follows by Theorem in Sec 22

that $f'(z)$ is analytic everywhere, and

$$f''(\underset{x+iy}{\overset{\shortparallel}{z}}) = (f'(z))' = \tilde{u}_x(x,y) + i\tilde{v}_x(x,y) =$$

$$= -\cos(x)\cosh(y) + i\sin(x)\sinh(y)$$

$$= -f(z).$$