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$$\cos(z) = 2$$

$$\frac{e^{iz} + e^{-iz}}{2}, \quad z = x + iy$$

$$\text{Set } w = e^{iz}, \text{ Then } \frac{w + \frac{1}{w}}{2} = 2 \Leftrightarrow w^2 - 4w + 1 = 0$$

$$w_{1,2} = 2 \pm \sqrt{3}$$

$$\text{Write } z = x + iy. \text{ So } e^{iz} = e^{-y + ix}$$

$$\text{So } e^{-y + ix} = 2 \pm \sqrt{3}, \text{ which is real and positive.}$$
$$e^{-y} e^{ix}$$

Now $e^{-y} > 0$. Hence e^{ix} is real and positive and is hence $= 1$. So $x = 2k\pi$, k integer.

$$e^{-y} = 2 + \sqrt{3} \text{ or } 2 - \sqrt{3}. \text{ Both are positive.}$$

$$\text{So } -y = \ln(2 + \sqrt{3}) \text{ or } \ln(2 - \sqrt{3})$$

$$(2 + \sqrt{3})(2 - \sqrt{3}) = 1.$$

$$\text{So } \ln(2 - \sqrt{3}) = -\ln(2 + \sqrt{3}),$$

$$\text{Hence, } y = \pm \ln(2 + \sqrt{3})$$

$$\text{Hence, } z = 2k\pi \pm i \ln(2 + \sqrt{3}), \quad k \text{ integer.}$$