where $k \in \mathbb{Z}$

Sec 33, page 104 #7

$$i^{a+bi} = e^{\log(i)\cdot(a+bi)} = e^{\left(\frac{\pi i}{2}+2k\pi i\right)(a+bi)} =$$

$$= e^{-\left(\frac{\pi b}{2}+2k\pi b\right)} \cdot \underbrace{e^{2k\pi a i}}_{\text{abs value is } 1}$$

So $\left|i^{a+bi}\right| = \left\{ e^{-\left(\frac{\pi b}{2}+2k\pi b\right)} ; k \in \mathbb{Z} \right\}$

The function $f(x)=e^x$, $f: \mathbb{R} \to \mathbb{R}$ is one-to-one and the set $\left\{ -\left(\frac{\pi b}{2}+2k\pi b\right) ; k \in \mathbb{Z} \right\}$ is infinite

if $b \neq 0$, so the set of all possible values of $\left|i^{a+bi}\right|$ is infinite if $b \neq 0$.

If $b=0$ $\left|i^{a}\right| = \left|e^{2k\pi a i}\right| = 1$.

Sec 33 page 104 #9 :

If f is analytic, and $c \neq 0$, $c \in \mathbb{C}$,

then $\frac{\partial}{\partial z} c^{f(z)} = \frac{\partial}{\partial z}\left(e^{\log(c) \cdot f(z)}\right) =$

$= \log(c)\, e^{\log(c) \cdot f(z)} \cdot f'(z)$

$= \log(c)\, c^{f(z)} \cdot f'(z)$,

where $\log(z)$ is a branch of log

Sec 26 page 81 # 1(a)

Show that $u(x,y) = 2x - 2xy$ is Harmonic (on $\mathbb{R}^2$) and find a harmonic conjugate.

Answer:

$u_x = 2 - 2y$ $\qquad$ $u_y = -2x$

$u_{xx} = 0$ $\qquad$ $u_{yy} = 0$

The first and second partials are continuous and $u_{xx} + u_{yy} \equiv 0$, and so u is Harmonic.

$$v(x,y) = \int \underbrace{v_y(x,y)}_{\overset{\text{Cauchy Riem}}{=} u_x(x,y)} dy = \int (2-2y)dy = 2y - y^2 + h(x).$$

$$\left.\begin{array}{l} v_x = h'(x) \\ v_x = -u_y = +2x. \end{array}\right\} \Rightarrow \begin{array}{l} h'(x) = +2x \\ h(x) = -x^2 + C_1. \end{array}$$

So $v(x,y) = 2y - y^2 + x^2 + C_1$.

Note that

$$\boxed{u + iv = 2z + iz^2 + C_1}$$