Section 31 page 97 # 5(b)

The set of values of $\log(\underbrace{i^2}_{-1})$ is

$$\underbrace{\ln\underbrace{|-1|}_{1}}_{0} + i\arg(-1) = \{ i(\pi + 2k\pi) ; k \in \mathbb{Z} \}.$$

The set of values of $\log(i)$ is

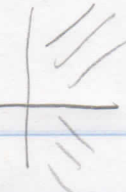
$$\underbrace{\ln\underbrace{|i|}_{1}}_{0} + i\left(\frac{\pi}{2} + 2k\pi\right), \quad k \in \mathbb{Z}.$$

So the set of values of $2\log(i)$ is

$$\{ \pi i + 4k\pi i ; k \in \mathbb{Z} \}$$

The two sets are indeed different.

Sec 32 page 100 problem 1:



If $Re(z_i) > 0$, the $-\frac{\pi}{2} < Arg(z_i) < \frac{\pi}{2}$, $i=1,2$,

So $-\pi < Arg(z_1) + Arg(z_2) < \pi$ is a principal argument. Now,

(i) $e^{[Log(z_1) + Log(z_2)]} = e^{Log(z_1)} \cdot e^{Log(z_2)} = z_1 \cdot z_2$,

and

(ii) and $Im(Log(z_1) + Log(z_2)) = Arg(z_1) + Arg(z_2)$ belongs to $(-\pi, \pi]$. By definition, $Log(z_1 z_2)$ is the uni complex number satisfying (1) $e^{Log(z_1 z_2)} = z_1 z_2$ and

(2) $Im(Log(z_1 z_2))$ belongs to $(-\pi, \pi]$,

Hence, $Log(z_1 z_2) = Log(z_1) + Log(z_2)$,