

Additional problem for Section 5B:

Let f be an entire function.

Suppose that $|f(z)| > \frac{1}{2}$ for all $z \in \mathbb{C}$,
(*)

Set $g(z) := \frac{1}{f(z)}$. Then g is entire,

since $f(z)$ is nowhere zero, by (*), and $f(z)$ is entire. Now,

$$|g(z)| = \frac{1}{|f(z)|} < \frac{1}{\frac{1}{2}} = 2. \quad \text{Hence, } g \text{ is}$$

by (*)

entire and bounded, and so constant by Liouville's Theorem.