

Section 79 page 239 Problem 1 parts (a) and (c)
Residue at $z=0$:

$$1(a) \quad \frac{1}{z+z^2} = \frac{1}{z} \cdot \frac{1}{1+z} = \frac{1}{z} \left(1 - z + z^2 - \dots \right)$$

$$\underbrace{\frac{1}{1+z}}_{\sum_{m=0}^{\infty} (-1)^m z^m}$$

So the coefficient of $\frac{1}{z}$ in the Laurent series in the punctured disk $\{z : 0 < |z| < 1\}$ is 1

$$\boxed{\operatorname{Res}_{z=0} \frac{1}{z+z^2} = 1}$$

$$1(c) \quad f(z) = \frac{z - \sin(z)}{z} = \frac{1}{z} \left(\frac{z^3}{3!} - \frac{z^5}{5!} + \dots \right)$$

$$\uparrow$$

$$\sin(z) = \sum_{m=0}^{\infty} \frac{(-1)^m z^{2m+1}}{(2m+1)!} = z - \frac{z^3}{3!} + \dots$$

$$= \frac{z^2}{3!} - \frac{z^4}{5!} + \dots$$

The coeff of $\frac{1}{z}$ is zero,

$$\boxed{\operatorname{Res}_{z=0} f(z) = 0}$$