Problem 4 page 188 Sec 56:

$z = re^{i\theta}$, $\quad 0 < r < 1$,

$$\sum_{n=1}^{\infty} \underbrace{z^n}_{= r^n e^{in\theta}} = \frac{z}{1-z} = \frac{re^{i\theta}}{1-re^{i\theta}} = \frac{\overbrace{re^{i\theta}(1-re^{-i\theta})}^{re^{i\theta} - r^2}}{\underbrace{(1-re^{i\theta})(1-re^{-i\theta})}_{1-2r\cos(\theta)+r^2}}$$

$$= \frac{(r\cos(\theta) - r^2) + i\, r\sin(\theta)}{1 - 2r\cos(\theta) + r^2}.$$

Taking the real part of both sides, and using the Theorem of Sec 56, we get

$$\sum_{n=1}^{\infty} r^n \cos(n\theta) = \frac{r\cos(\theta) - r^2}{1 - 2r\cos(\theta) + r^2}$$

Taking the imaginary part of both sides, and using the Theorem of Sec 56, we get

$$\sum_{r=1}^{\infty} r^n \sin(n\theta) = \frac{r\sin(\theta)}{1 - 2r\cos(\theta) + r^2}$$