Sec 54 page 179 #3: R closed and bounded region.

f analytic on the interior of R (a domain D), continuous on R, and non-constant. Furthermore, $f(z) \neq 0$ for all z in R.

The function $|f(z)|$ is continuous on R, being the composition of the absolute value function and f, both continuous. The function $1/|f(z)|$ is continuous as well, since $|f(z)|$ does not vanish on R. So $1/|f(z)|$ has a maximum on R at z_0 if and only if $|f(z)|$ has a minimum at z_0.

Set $g(z) = 1/f(z)$. Then $g(z)$ is continuous on R, analytic and non-constant on D, and so $|g(z)|$ obtains its maximum on the boundary of R and not in D by the Corollary of Sec 54. Hence, $|f(z)|$ obtains its minimum on the boundary of R and not in its interior D.

Q.E.D

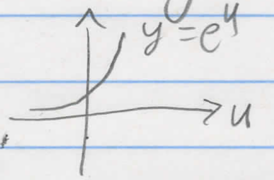
Section 54 page 179 #6: Assume

$f(z) = u(x,y) + i v(x,y)$ is continuous on the closed and bounded region R and not constant throughout its interior (a domain D).

Let $g(z) = e^{f(z)}$. Then $g(z)$ is continuous on R and not constant on D. Furthermore $|g(z)| = e^{u(z)}$ does not vanish on R.

By problem 3 in Sec 54, $g(z)$ has a minimum value in R, which occurs on the boundary of R and NOT in its interior D.

Now $h(u) = e^u$ is a strictly increasing function on $\mathbb{R}$ (reals).



So a minimum of $|e^{u(z)}|$ occurs at $z_0 = x_0 + iy_0$ if and only if z_0 is a minimum of $u(x,y)$. Thus $u(x,y)$ has its minimum at a boundary point of R, and never in its interior D.

Q.E.D