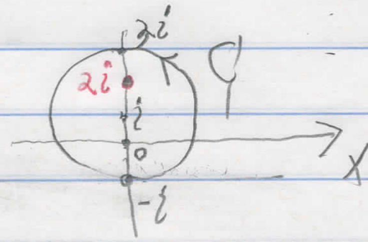
Section 52 page 170 #2

C is the circle of radius 2 centered at i



a) $g(z) = \dfrac{1}{z^2+4} = \dfrac{1}{(z+2i)(z-2i)}$

$$\int_C g(z)\,dz = \int_C \frac{(1/z+2i)}{z-2i}\,dz =$$

Cauchy's Integral Formula.

$$= 2\pi i \, \frac{1}{2i+2i} = \frac{\pi}{2}.$$

b) $g(z) = \dfrac{1}{(z^2+4)^2} = \Big[\underbrace{\dfrac{1}{(z+2i)^2}}_{f(z)}\Big] \Big/ (z-2i)^2$

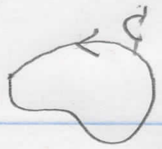
$$\int_C g(z)\,dz = (2\pi i)\, f'(2i) =$$

Cauchy's Integral Formula for the first derivative

$f'(z) = -2(z+2i)^{-3}$

$$= 2\pi i \cdot (-2)\,\underbrace{(4i)^{-3}}_{\frac{1}{4^3}\cdot i} = \frac{\pi}{16}.$$

Section 52 page 170 #5:



Let f be analytic within and on a simple closed contour C and z_0 is not on C. We need to prove the equality

$$\int_C \frac{f'(z)}{z-z_0}\,dz \overset{\circledast}{=} \int_C \frac{f(z)}{(z-z_0)^2}\,dz.$$

f has derivatives of all order at points where it is analytic, in particular f' is analytic as well (by Theorem 1 in Sec 52).

- If z_0 is in the unbounded domain "outside" C, then both $\frac{f'(z)}{z-z_0}$ and $\frac{f(z)}{(z-z_0)^2}$ are analytic within and on C, and so both sides of $\circledast$ vanish, by Cauchy-Goursat.
- If z_0 is within C, then the LHS of $\circledast$ is equal to $(2\pi i)f'(z_0)$, by Cauchy's Integral Formula, and the RHS of $\circledast$ is equal to $(2\pi i)f'(z_0)$, by Cauchy's Integral formula for the first derivative.