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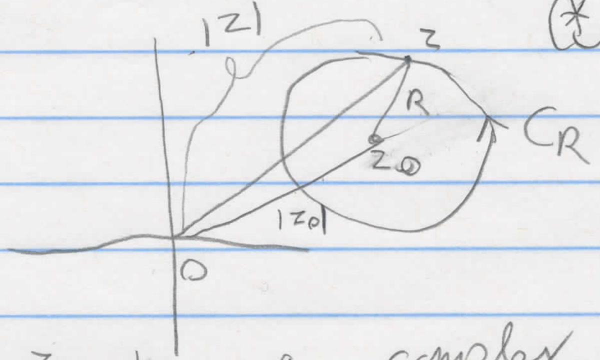
f entire and $|f(z)| \le A|z|$ for all $z \in \mathbb{C}$. (*)



Let z_0 be a complex number and C_R the circle of radius R center at z_0 and oriented counterclockwise. If z is on C_R, then $|z| = |z - z_0 + z_0| \le \underbrace{|z-z_0|}_{R} + |z_0| = R + |z_0|$.

So $|f(z)| \le A|z| \le \underbrace{A(R+|z_0|)}_{M_R}$. Hence, by Cauchy's Inequality

$$0 \le |f^{(2)}(z_0)| \le \frac{2! \cdot M_R}{R^2} = \frac{2A(R+|z_0|)}{R^2} \xrightarrow{R \to \infty} 0.$$

Hence $f''(z_0) = 0$, for all z_0.
Hence, $f'(z) = a_1$, for some constant complex number a_1. Hence
$f(z) = a_1 z + a_0$, for some complex number a_0.

But $\underset{|a_0|}{\underset{\shortparallel}{|f(0)|}} \le A|0| = 0$. Hence $a_0 = 0$
by (*)

And $f(z) = a_1 z$.