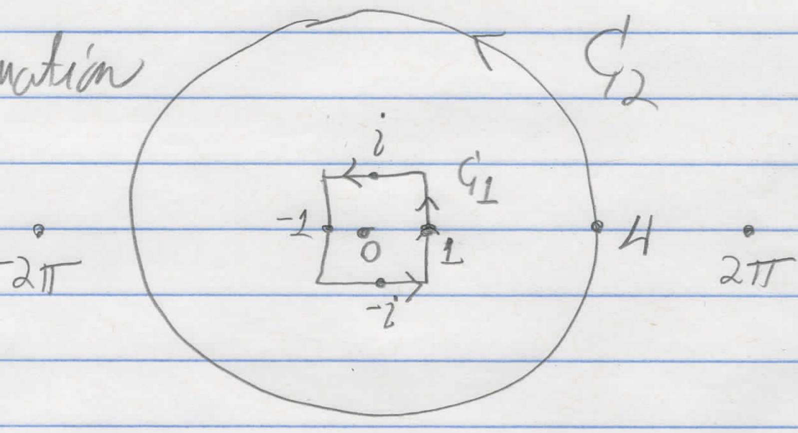
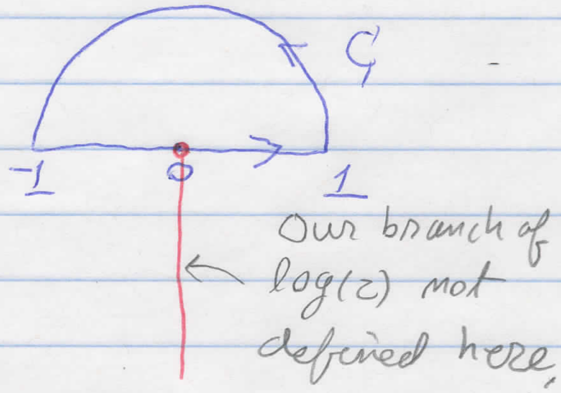
Sec 45 page 160 problem 2b:

$f(z) = \dfrac{z+2}{\sin(z/2)}$ is analytic at every point other than the zeroes of $\sin(z/2)$. The zeroes of $\sin(w)$ are all real at $w = k\pi$, $k \in \mathbb{Z}$. Hence, the zeroes of $\sin(z/2)$ are at $z \in \{2k\pi : k \in \mathbb{Z}\}$. Now, if $k \neq 0$, then $|2k\pi| > 4$, and $2k\pi$ is not enclosed by C_2. If $k=0$, then the origin __is__ enclosed by C_1. Thus, f is analytic at all points on C_1 and C_2 and at all points which are enclosed by C_2 but not by C_1. The equality $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$ thus follows from the Principle of Deformation of the Paths in Sec 49.



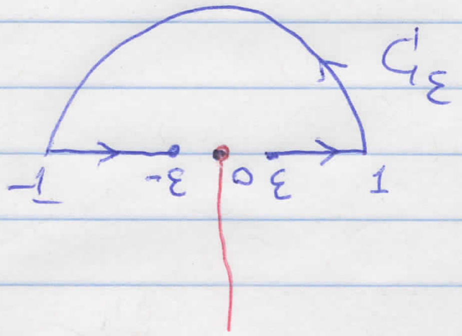
6) Section 49 page 160 problem 6:



Let $\log(z) := \ln|z| + i\arg(z)$, where $\arg$ is the branch of the argument function with values in $(-\frac{\pi}{2}, \frac{3\pi}{2})$.

The function
$$\underset{x+iy}{f(z)} := \begin{cases} 0 & \text{if } z=0 \\ e^{\frac{1}{2}\log(z)} & \text{if } x \neq 0 \text{ or } y>0 \end{cases}$$

is the one given in the question. This function is continuous on C_1 and analytic at all points within and on C_1 EXCEPT at the origin $z=0$. Hence, we can not use Cauchy-Goursat. Instead, let C_ε be the contour obtained by going along C_1 from $+\varepsilon$ to $-\varepsilon$, where ε is a real number, $\varepsilon \in (0,1)$. Then



$$\int_{C_1} f(z)\,dz = \lim_{\varepsilon \to 0} \int_{C_\varepsilon} f(z)\,dz \overset{\uparrow}{=}$$

antiderive of $f(z)$ is $F(z) = \frac{2}{3} e^{3/2 \log(z)}$

$$= \frac{2}{3} \lim_{\varepsilon \to 0} \Big[e^{\frac{3}{2}\log(z)} \Big]_{-\varepsilon}^{\varepsilon} = \frac{2}{3} \lim_{\varepsilon \to 0} \left(\varepsilon^{3/2} - e^{3/2(\ln(\varepsilon) + \pi i)} \right) =$$

$$= \lim_{\varepsilon \to 0} \Big(\varepsilon^{3/2} + \varepsilon^{3/2} \cdot \underbrace{e^{\frac{3\pi i}{2}}}_{-i} \Big) = 0.$$