Sec 42 Problem 1 page 135:

$$f(z) = \frac{z+2}{z} = 1 + \frac{2}{z}, \quad z = 2e^{i\theta}, \quad a \le \theta \le b$$ a contour C. Then

$$\int_C f(z)\,dz = \int_a^b \left(1 + \frac{2}{2e^{i\theta}}\right) \underbrace{\frac{dz}{d\theta}}_{2ie^{i\theta}}\, d\theta =$$

$$\int_a^b \left[2ie^{i\theta} + 2i\right] d\theta = 2\left[e^{i\theta} + i\theta\right]_a^b =$$

$$= 2\left\{\left(e^{ib} + ib\right) - \left(e^{ia} + ia\right)\right\}.$$

(a) If $a = 0$, $b = \pi$ we get

$$\int_C f(z)\,dz = 2\left\{\left(\underbrace{e^{\pi i}}_{-1} + i\pi\right) - \left(\underbrace{e^{0}}_{1} + 0\right)\right\} =$$

$$2\left\{\pi i - 2\right\} = 2\pi i - 4.$$

(b) If $a = \pi$, $b = 2\pi$,

$$\int_C f(z)\,dz = 2\left\{\left(\underbrace{e^{2\pi i}}_{1} + 2\pi i\right) - \left(\underbrace{e^{i\pi}}_{-1} + i\pi\right)\right\} =$$

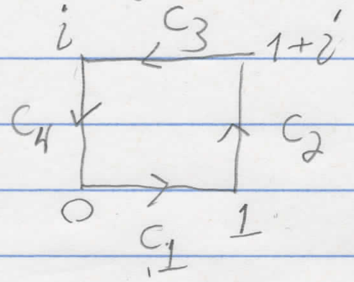
$$= 2\left\{2 + \pi i\right\} = 2\pi i + 4$$

(c) If $a = 0$, $b = 2\pi$,

$$\int_C f(z)\,dz = 2\left\{\left(\underbrace{e^{2\pi i}}_{1} + 2\pi i\right) - \left(e^{0} + 0\right)\right\} = 4\pi i.$$

Sec 42 Prob 3 page 135

$f(z) = \pi e^{\pi \bar{z}}$



$C = C_1 + C_2 + C_3 + C_4$

- C_1 is given by $z(t) = t$, $0 \le t \le 1$

$$\int_{C_1} f(z)\,dz = \int_0^1 \pi e^{\pi t} \cdot \underbrace{\frac{dz}{dt}}_{1} dt = \left[e^{\pi t}\right]_0^1 = e^{\pi} - 1$$

- C_2 is given by $z(t) = 1 + ti$, $0 \le t \le 1$

$$\int_{C_2} f(z)\,dz = \int_0^1 \underbrace{\pi e^{\pi(1-ti)} \cdot \underbrace{\frac{dz}{dt}}_{i}}_{-\frac{d}{dt} e^{\pi(1-ti)}} dt = \left[-e^{\pi(1-ti)}\right]_0^1 =$$

$$= -\underbrace{e^{\pi(1-i)}}_{-e^{\pi}} + e^{\pi} = 2e^{\pi}$$

- C_3 is given by $z(t) = (1+i) - t$, $0 \le t \le 1$

$$\int_{C_3} f(z)\,dz = \int_0^1 \pi \underbrace{e^{\pi[(1-t)-i]}}_{-e^{\pi(1-t)}} \underbrace{\frac{dz}{dt}}_{-1} dt =$$

$$\int_0^1 \pi e^{\pi(1-t)} dt = \left[-e^{\pi(1-t)}\right]_0^1 = -e^{0} + e^{\pi} = e^{\pi} - 1.$$

C_4 is given by $z(t) = (1-t)i \quad 0 \le t \le 1$,

$$\int_{C_4} f(z)\,dz = \int_0^1 \underbrace{\pi e^{\pi(t-1)i} \underbrace{\frac{\partial z}{\partial t}}_{-i}}_{-\frac{\partial}{\partial t} e^{\pi(t-1)i}} dt =$$

$$= -\left[e^{\pi(t-1)i}\right]_0^1 = -\left\{\underbrace{e^0 - \underbrace{e^{-\pi i}}_{-1}}_{2}\right\} = -2$$

So $\displaystyle\int_C f(z)\,dz = \sum_{i=1}^{4} \int_{C_i} f(z)\,dz =$

$(e^{\pi}-1) + 2e^{\pi} + (e^{\pi}-1) + (-2) = 4e^{\pi} - 4$

Sec 42 Prob 5 page 135:

C a contour from z_1 to z_2

$f(z) = 1$. $\quad f(z) = F'(z)$ where $F(z) = z$,

So $\displaystyle\int_C f(z)\,dz = F(z_2) - F(z_1) = z_2 - z_1$.

See Sec 42 Example 3 or Sec 44.

Sec 42 Problem 10 page 136:

C_0 given by $z = z_0 + Re^{i\theta}$, $-\pi \le \theta \le \pi$.
C_1 " " $z = Re^{i\theta}$, " .

(a) $$\int_{C_0} f(z-z_0)\,dz \overset{def}{=} \int_{-\pi}^{\pi} f\Big(\underbrace{(z_0+Re^{i\theta})-z_0}_{= Re^{i\theta}}\Big)\, iRe^{i\theta}\,d\theta$$

$$= \int_{-\pi}^{\pi} f(Re^{i\theta})\, iRe^{i\theta}\,d\theta \underset{def}{=} \int_{C} f(z)\,dz.$$

(b) Let $f(z) = z^{n-1}$. Then

$$\int_{C_0} (z-z_0)^{n-1}\,dz = \int_{C_0} f(z-z_0)\,dz \underset{\text{Part a}}{=} \int_{C_1} f(z)\,dz =$$

$$= \int_{C} z^{n-1}\,dz = \begin{cases} 0 & \text{if } n \ne 0 \\ 2\pi i & \text{if } n = 0. \end{cases}$$

Eq (5) and (6) in Sec 42.