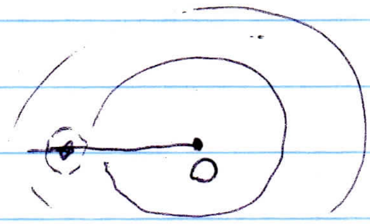
Sec 11, page 33

4 (a). The set



$S = \{z \mid -\pi < \arg(z) < \pi\}$ is $\mathbb{C}$ minus the non-positive part of the x-axis. Each open disk centered at $(x_0, 0)$, $x_0 \le 0$, contains a point in S, ~~Hence the~~ and is hence a boundary point of S. Hence, the closure of S is the whole of $\mathbb{C}$.

4 (b)

$S = \{z \mid |Re(z)| < |z|\} = \{z = x+iy : y \neq 0\} =$

$\mathbb{C}$ minus y-axis.

The closure of S is the whole of $\mathbb{C}$, for reason similar to 4 (a).

Sec 12, page 37, # 4:

$$f(z) \underset{z = re^{i\theta}}{=} z + \frac{1}{z} = re^{i\theta} + \left(\frac{1}{r}\right) e^{-i\theta} =$$

$$= r(\cos(\theta) + i\sin(\theta)) + \frac{1}{r}(\cos(\theta) - i\sin(\theta)) =$$

$$= \left(r + \frac{1}{r}\right)\cos(\theta) + i\left(r - \frac{1}{r}\right)\sin(\theta)$$

Sec 18 page 55 Problem 5:

$$f(z) = \left(\frac{z}{\bar{z}}\right)^2$$

$$\lim_{(x,0)\to(0,0)} \left(\frac{x+0i}{\overline{x+0i}}\right)^2 = \lim_{x\to 0}\left(\frac{x^2}{x^2}\right) = \lim_{x\to 0} 1 = 1.$$

$$\lim_{(x,x)\to(0,0)} \left(\frac{x+ix}{x-ix}\right)^2 = \lim_{x\to 0}\left(\frac{|x|\sqrt{2}\, e^{\pi i/4}}{|x|\sqrt{2}\, e^{-\pi i/4}}\right)^2 =$$

$$= \lim_{x\to 0} \underbrace{\Big(\underbrace{e^{\pi i/2}}_{i}\Big)^2}_{-1} = -1.$$

Since $1 \neq (-1)$, $\lim_{z\to 0} f(z)$ does not exist.

Sec 20 page 62; #9

$$f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0, \end{cases}$$

$$f'(0) = \lim_{\Delta z \to 0} \frac{f(0+\Delta z) - \overbrace{f(0)}^{0}}{\Delta z} =$$

$$= \lim_{\Delta z \to 0} \frac{\left(\dfrac{(\overline{\Delta z})^2}{\Delta z}\right)}{\Delta z} = \lim_{\Delta z \to 0} \frac{(\overline{\Delta z})^2}{(\Delta z)^2} =$$

$$= \lim_{z \to 0} \frac{(\bar{z})^2}{z^2}$$

$$\lim_{(x,0)\to(0,0)} \frac{(\bar{x})^2}{x^2} = \lim_{x \to 0} 1 = 1$$

$$\lim_{y \to 0} \frac{(\overline{iy})^2}{(iy)^2} = \lim_{y \to 0} \frac{(-iy)^2}{(iy)^2} = \lim_{y \to 0} 1 = 1$$

$$\lim_{x+ix \to 0} \frac{(x-ix)^2}{(x+ix)^2} = \lim_{x \to 0} \frac{\left(|x|\sqrt{2}\, e^{-\pi i/4}\right)^2}{\left(|x|\sqrt{2}\, e^{\pi i/4}\right)^2} =$$

$$= \lim_{x \to 0} \Big(\underbrace{e^{-\pi i/2}}_{-i}\Big)^2 = \lim_{x \to 0} (-1) = -1.$$

Since $1 \neq -1$, the derivative limit does not exist.