Math 421 HW 1

Sec 3 page 8 1 (b):

$$\frac{5i}{\underbrace{(1-i)(2-i)}_{1+i(-2-1)}(3-i)} = \frac{5i}{-i\underbrace{(3-i)(3+i)}_{9+1}} = -\frac{1}{2}$$

$$1-3i = i(-3-i)$$

Sec 8 page 22 #9:

$(1+z+\cdots+z^n)(1-z) = 1-z^{n+1}$. Hence, if $z\neq 1$, dividing both sides by $1-z$ we get

5 pt

$$1+z+z^2+\cdots+z^n = \frac{1-z^{n+1}}{1-z}$$

substitute $e^{i\theta} = \cos(\theta)+i\sin(\theta)$, $0<\theta<2\pi$ for z and take the real part of both sides to get

5 pt

$$1+\cos(\theta)+\underbrace{\cos(2\theta)}_{\substack{= Re((e^{i\theta})^2) \\ e^{2\theta i}}}+\cdots+\cos(n\theta) = Re\left(\frac{1-e^{(n+1)\theta i}}{1-e^{i\theta}}\right)$$

5 pt

Now, $$\frac{1-e^{(n+1)\theta i}}{1-e^{i\theta}} = \frac{e^{-\frac{i\theta}{2}} - e^{\overbrace{(n+\frac{1}{2})}^{\frac{2n+1}{2}}\theta i}}{e^{-i\theta/2} - e^{i\theta/2}} = \frac{[\cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})] - [\quad]}{-2i\sin(\theta/2)}$$

$\cos(\ ) + i\sin(\ )$

Multiply numerator and denominator by $e^{-i\theta/2}$

$$= \frac{\sin(\frac{\theta}{2}) + i\cos(\frac{\theta}{2}) + \sin(\frac{(2n+1)}{2}\theta) - i\cos(\quad)}{2\sin(\theta/2)}$$

Taking the real part we get

$$Re\left(\frac{1-e^{(n+1)\theta i}}{1-e^{i\theta}}\right) = \frac{1}{2} + \frac{\sin(\frac{2n+1}{2}\theta)}{2\sin(\theta/2)}.$$

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Extra problem for section 5

Use established properties of moduli to show that when $|z_3| \neq |z_4|$, then

$$\left|\frac{z_1 + z_2}{z_3 + z_4}\right| \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||} \quad (*)$$

Solution: $|z_1 + z_2| \leq |z_1| + |z_2|$, by the triangle inequality.

$|z_3 + z_4| \geq ||z_3| - |z_4|| > 0$.

$\boxed{\text{by Eq (8) page 11}}$

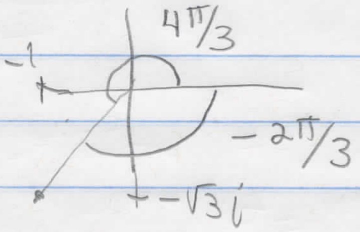
since $|z_3| \neq |z_4|$

Hence, inequality $(*)$ follows.

Sec 10 page 29: 2 (b)

Fourth roots of

$$-8-8\sqrt{3}\,i = 8\underbrace{\left(-1-\sqrt{3}\,i\right)}_{2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)} = 2^4\cdot e^{4\pi/3\, i}$$



The principal root is $w := 2\,e^{\frac{\pi}{3}i} =$

$$= 2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = 1+\sqrt{3}\,i$$

The three additional roots are

$w\cdot i = -\sqrt{3}+i$

$w\cdot(-1) = -1-\sqrt{3}\,i$

$w\cdot(-i) = \sqrt{3}-i.$