1. Let $f(z) = \frac{g(z)}{h(z)}$, where g and h are analytic at z_0, $g(z_0) \neq 0$, $h(z_0) = 0$, and $h'(z_0) \neq 0$. Show that

$$Res_{z=z_0}(f) = \frac{g(z_0)}{h'(z_0)}. \tag{1}$$

   Hint: Show first the equality $Res_{z=z_0}(f) = \lim_{z \to z_0}(z - z_0)f(z)$.

2. Use Equation (1) to find the residue of $\tan(z)$ at $\frac{\pi}{2}$.

3. Consider the Laurent series

$$\tan(z) = \sum_{n=-\infty}^{\infty} a_n z^n$$

   that is valid in the annulus $\{z \in \mathbb{C} \mid \frac{\pi}{2} < |z| < \frac{3\pi}{2}\}$. Use contour integrals to show that the coefficients a_n with indices in the range $-\infty < n \leq -1$ satisfies:

$$a_n = \begin{cases} -2\left(\frac{\pi}{2}\right)^{k-1} & \text{if } \quad k \text{ is } \textit{odd}, \\ 0 & \text{if } \quad k \text{ is } \textit{even}. \end{cases}$$