MATH 300 FINAL REVIEW, FALL 2015

Chapter 3:

1. Solve the linear congruences:
(a) $4x \equiv 6 \pmod{14}$ (b) $29x \equiv 62 \pmod{128}$.
2. Construct the addition and multiplication tables for $\mathbb{Z}_6$.
3. Suppose the 9-digit number $1234x6789$ is divisible by 9. Find all possible values of x.
4. Solve for $x \in \mathbb{Z}$: $x^6 \equiv 6x \pmod{7}$.
5. Find the multiplicative inverse of $[3]$ in $\mathbb{Z}_{41}$.
6. Find the remainder when 14^{181} is divided by 99.
7. If $2p^2 = q^3$, where $p, q \in \mathbb{Z}$, show that 2 is a common divisor of p and q.
8. Show that an integer of the form $7m + 5$ can not be a perfect square.
9. Show that $\sqrt{6}$ is not a rational number.

The Chinese Remainder Theorem.

10. Solve the simultaneous congruences:
(i) $x \equiv 5 \pmod{7}$, $x \equiv 23 \pmod{25}$.
(ii) $2x \equiv 11 \pmod{13}$, $3x \equiv 7 \pmod{8}$, $7x \equiv 5 \pmod{9}$.
11. Find the last two digits of 556^{3333}.
12. Show that if p, q are integers, not divisible by 3 or 5, then $p^4 \equiv q^4 \pmod{15}$.
13. Solve for $x \in \mathbb{Z}$: $x^2 \equiv 4 \pmod{30}$.
14. Solve for $x \in \mathbb{Z}$: $x^{32} + x + 1 \equiv 0 \pmod{35}$.

Relations and Equivalence Relations.

15. Determine whether the following relations on $\mathbb{Z}$ are reflexive, symmetric, or transitive.
(a) aRb if and only if $a + b \neq 1$.
(b) aRb if and only if $a - b \geq 0$.
(c) aRb if and only if $a \neq b$.
16. Show that the following relation is an equivalence relation: for any real numbers a and b, aRb if and only if $a - b = 2\pi k$ where $k \in \mathbb{Z}$.

Chapter 6: Functions, Cardinality

17. Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by the formula $f(x) = \sqrt{x^2 + 1}$.
(a) What is the domain of f?
(b) What is the codomain of f?
(c) What is the image of f?
(d) Find maximal subsets X, Y of $\mathbb{R}$ such that $f : X \to Y$ is a bijection. Find a formula for the inverse $f^{-1} : Y \to X$.

18. Let $f : \mathbb{Z}_6 \to \mathbb{Z}_6$ be the function $f([x]) = [x^2]$ and let $g : \mathbb{Z}_6 \to \mathbb{Z}_6$ be the function $g([x]) = [2x + 1]$. Find a formula and construct a table for $f \circ g$. Sketch the graph of $f \circ g$ and decide whether it is injective or surjective.

19. Show that $\#(\mathbb{P}_3 \times \mathbb{P}) = \#\mathbb{P}$ by constructing an explicit bijection $f : \mathbb{P}_3 \times \mathbb{P} \to \mathbb{P}$.

20. Let X be the set consisting of integers which have remainder 1 when divided by 3. Show that $\#X = \#\mathbb{Z}$.