Math 300.2 Quiz 3 Spring 2016

Name: Solution

Prove, by induction, the equality $\underbrace{1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1)}_{f(n) \overset{(*)}{=} \sum_{i=1}^{n} i(i+1)} \overset{(**)}{=} \frac{n(n+1)(n+2)}{3}$.

Case $n=1$: $f(1) = 1(1+1) = 2$, $\frac{1(1+1)(1+2)}{3} = 2$, O.K.

INDUCTION STEP:

Assume that $f(n) = \frac{n(n+1)(n+2)}{3}$.

We need to show that $f(n+1) = \frac{(n+1)(n+2)(n+3)}{3}$

By its definition (*), $f(n+1) = f(n) + (n+1)((n+1)+1) =$

By the Ind. Hyp.

$$\overset{\downarrow}{=} \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) =$$

$$\frac{\boxed{n}(n+1)(n+2) + \boxed{3}(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}.$$

Hence, equation (**) holds for all n, by the Induction Principle. Q.E.D.