Math 300.2 Quiz 2 Spring 2016

Name: Solution

Let a, b, c be non-zero integers, such that $c > 0$.

1. (60 points) Prove that $c\gcd(a, b)$ is a common divisor of ac and bc. Please denote $\gcd(a, b)$ by d in your proof.

Let $d := \gcd(a,b)$. Then there exist integers x, y such that $a = dx$ and $b = dy$, since d is a common divisor.

So $ac = dxc = (cd)x$ and $bc = dyc = (cd)y$.

Hence, cd is a common divisor of ac and bc (by definition of a divisor).

Def: An integer d divides an integer m, if there exists an integer q, such that $m = dq$.

2. (40 points) Prove the inequality $c\gcd(a, b) \leq \gcd(ac, bc)$.

We have shown in 1 that $c\gcd(a,b)$ is a common divisor of ac and bc. Any common divisor is $\leq$ the greatest common divisor, by definition of gcd.