Math 300 Section 2 Midterm 1 Spring 2016

Name:_____

1. (15 points) Let P and Q be statements. Find the truth tables of the following statements and use them to determine if they are equivalent.
   i) $P \Rightarrow Q$.
   ii) (NOT Q) $\Rightarrow$ (NOT P).

2. (10 points) Let the universe of discourse be the real numbers. Prove or give a counter example to the following statement.
   $\forall x \; \exists y \; (x^2 > y^2)$.

3. (10 points) Let the universe of discourse be the real numbers. Write the contrapositive of the following statement.
   If $x < -2$, then $x^2 > 4$.

4. (15 points) Let S and T be sets. Use the contrapositive method to prove the following statement.
   $((S \cap T = \emptyset)$ AND $(S \cup T = T)) \Rightarrow (S = \emptyset)$.
   Suggestion: State the equivalent contrapositive statement and start your argument with: "Assume that $S \neq \emptyset$. Then there exists an element $x \in S$. ..."

5. (10 points) How many positive divisors does 360^3 has? Justify your answer!

6. (10 points) Let x, y, z, and w be positive integers. Assume that $\gcd(x, z) = 1$ and that $y|z$. Prove that if $x|yw$, then $x|w$.

7. (15 points) a) Use the Extended Euclidean Algorithm to find a particular solution of the equation $33x + 18y = \gcd(33, 18)$.

   b) Find all the integer solutions of the equation $33x + 18y = 150$. Show all your work.

   c) Find all positive integer solutions of the equation $33x + 18y = 150$.

8. (15 points) a) Let x be a positive integer and $x = p_1^{d_1} p_2^{d_2} \cdots p_n^{d_n}$ its prime decomposition. What can you say about the parity (even or odd) of the exponents in the prime decomposition of x^2?

   b) Let p be a prime. Prove, by contradiction, that the equation $px^2 = y^2$ does not have any positive integer solutions x, y. Hint: Use part **??**a.

   c) (Bonus 5 points) Prove, by contradiction, that if p is prime then $\sqrt{p}$ is not a rational number.