MATH 300 MIDTERM FALL 2015

There are totally 9 problems in this exam.

Show all your work !!!

1. (15 pts) Let P, Q be statements. Show that the statement NOT (P OR Q) is equivalent to the statement (NOT P) AND (NOT Q) by showing they have the same truth tables.

2. (10 pts) Prove or give a counterexample to the following statement:

$$\forall x \in \mathbb{Z}, \ \ 2x^3 + 2x + 5 \neq 0.$$

3. (10 pts) Prove, using the contrapositive method, that

$$\text{if } 2x^3 + 3x^2 + 5x - 10 \leq 0, \text{ then } x \leq 1.$$

4. (10 pts) Define the sequence x_n as follows:

$$x_1 = x_2 = 6, \text{ and for } n \geq 3, \ x_n = 2x_{n-1} + 3x_{n-2}.$$

Prove that for all $n \geq 1$, $x_n = 3(3^{n-1} + (-1)^{n-1})$.

5. (10 pts) Determine whether the following is true or not, and explain why.

NOT $(\forall x \in D, P(x) \Rightarrow Q(x))$ is equivalent to $\exists x \in D, ((\text{NOT } P(x)) \text{ AND } Q(x))$.

6. (10 pts) Find an expression for

$$S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n+1} n, \text{ where } n = 1, 2, 3, \cdots,$$

and prove the expression for S_n is correct.

7. (15 pts) Consider the linear Diophantine equation: $10x + 25y = 200$.
(1) Find all integer solutions of the equation.
(2) Find all non-negative integer solutions of the equation.

8. (10 pts) Determine whether 223 is prime or not, and explain why.

9. (10 pts) Let $u, v \in \mathbb{Z}$. Suppose $gcd(3, u) = 1$. Prove that if $u \mid 9v$, then $u \mid v$.