Fall 2015 MATH 300 FINAL EXAM NAME:

There are 10 problems in this exam.

Show all your work. Good luck !!!

1. This problem consists of 3 independent parts.
(1) (5 pts) Negate the expression: $\forall y \in \mathbb{R}$, $\exists x \in \mathbb{R}$, $(x^2 - y = 1) \Rightarrow (x \neq y)$.
(2) (5 pts) Disprove: $\forall x \in \mathbb{R}$, if $x^3 + 40x^2 - 41 \geq 0$, then $x > 1$.
(3) (5 pts) Let R be the relation on the set $\mathbb{Q}$ defined as follows: for any $x, y \in \mathbb{Q}$, xRy if $x - y \geq 0$. Is R an equivalence relation? Why?

2. (10 pts) Prove that $x^3 + 2x - 2 = 0$ has no integer solutions by contradiction.

3. (10 pts) Solve for $x \in \mathbb{Z}$: $2x^{33} + 5x + 1 \equiv 0 \pmod{34}$.

4. (10 pts) Find the last 2 digits of 423^{324}.

5. (10 pts) Solve linear congruence: $48x \equiv 20 \pmod{44}$.

6. (10 pts) Prove that there exists no $m \in \mathbb{Z}$ such that $7m + 6$ is a perfect square.

7. (10 pts) Show that the function $f : \mathbb{Z}_{11} \to \mathbb{Z}_{11}$, where $f([x]) = [6] \cdot [x]$, is a bijection. Furthermore, find a formula for the inverse $f^{-1} : \mathbb{Z}_{11} \to \mathbb{Z}_{11}$.

8. (5 pts) Use Mathematical Induction to show that for any $n \geq 1$,

$$1 + t + t^2 + \cdots + t^n = \frac{1 - t^{n+1}}{1 - t}.$$

9. (10 pts) Solve for $x \in \mathbb{Z} : x^3 \equiv 11 \pmod{30}$.

10. (10 pts) Prove that $\#\mathbb{P} = \#(\mathbb{P}_2 \times \mathbb{P})$ by constructing an explicit bijection (i.e., given by a formula) from $\mathbb{P}$ to $\mathbb{P}_2 \times \mathbb{P}$. Furthermore, find a formula for the inverse of your bijection.

See HW 13 Prob 11.5 for definition

See the homework 13 on Sec 6.9 (Permutations)

11. Prove, by induction, that every permutation of $\mathbb{P}_n$ can be written as a composition of transpositio