Math 300 Section 2 Midterm 2 Fall 2017

Name:____________________________

1. (15 points) Define the sequence x_n of rational numbers as follows. $x_1 = 1$, and

$$x_{n+1} = \left(\frac{n}{n+1}\right) x_n + 1, \text{ for all } n \geq 1.$$

Find an expression for x_n and prove, by induction, that the expression is correct.

2. (15 points) Let $f(x) = e^{3x}$ and denote by $f^{(n)}(x)$ its n-th derivative. Prove the following identity for all positive integers n.

$$\sum_{k=0}^{n} \binom{n}{k} 2^{n-k} f^{(k)}(x) = 5^n e^{3x}.$$

3. (15 points) Determine the number of congruence classes which solve the linear congruence $25x \equiv 35 \pmod{45}$ and find all of them. Justify your answer!

4. (15 points) Use the Extended Euclidean Algorithm (E.E.A) to find the inverse of $[80]$ in $\mathbb{Z}_{253}$. Credit will be given only for an answer using the E.E.A.

5. (15 points) Use the Chinese Remainder Theorem in order to determine (only) the **number** of congruence classes in $\mathbb{Z}_{65}$ solving the congruence

$$[x]^{14} + 12[x]^{12} \equiv [3].$$

You do not need to actually solve the congruence. Justify your answer.

6. (15 points) Find all integers x solving the simultaneous congruences

$$x \equiv 17 \pmod{41}, \tag{1}$$
$$x \equiv 20 \pmod{23}. \tag{2}$$

Justify your answer.

7. (15 points) Consider the relation R on the set of real numbers defined by xRy if and only if $x - y$ is an integer. Prove that R is an equivalence relation.