Math 300 Section 1 Midterm 1 Spring 2017

Name:__________________________

1. (15 points) Let P and Q be statements. Find the truth tables of the following statements and use them to determine if they are equivalent.
   i) $P \Rightarrow Q$.
   ii) (NOT P) OR (NOT Q) $\Rightarrow$ (NOT P).

2. (10 points) Let the universe of discourse be the real numbers. Prove the following statement: $\forall \epsilon > 0\ \exists \delta > 0,\ |x-2| < \delta \Rightarrow |3x-6| < \epsilon$.

3. (10 points) Let the universe of discourse be the real numbers. Write first the contrapositive and then the converse of the following statement.
If $x^2 + y^2 = 25$, then $|x| \leq 5$.

4. (15 points) Let S and T be sets. Prove or give a counter example.
$S \cup T = T \Leftrightarrow (S \subset T)$. Hint: Write the definition of $S \cup T$ in the form $S \cup T = \{x \ : \text{ some condition involving } x,\ S, \text{and } T\}$. Then use the membership condition in your answer.

5. (10 points) How many positive common divisors do 600 and 4500 have? Justify your answer!

6. (10 points) Prove that $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$ if and only if $\gcd(ab, c) = 1$.

7. (15 points) a) Use the Extended Euclidean Algorithm (E.E.A) to find a particular solution of the equation $57x + 12y = \gcd(57, 12)$. (Credit will be given only if the E.E.A is used).

b) Find all the integer solutions of the equation $57x + 12y = 300$. Show all your work.

c) Find all positive integer solutions of the equation $57x + 12y = 300$.

8. (15 points) Use induction to prove the following inequality for all positive integers n.
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^n} \geq 1 + \frac{n}{2}.$$
Hint: The left hand side is the sum $\sum_{k=1}^{2^n} \frac{1}{k}$ of 2^n terms, so when n is increased by 1 the number of terms gets multiplied by 2.