Math 235 Midterm 2 Spring 1999

1. (24 points) You are given below the matrix A together with its row reduced echelon form B

$$A = \begin{pmatrix} 1 & -1 & 0 & 3 & 0 & -1 \\ 2 & -2 & 1 & 8 & 0 & -1 \\ 1 & -1 & 1 & 5 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 0 & 3 & 0 & -1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

a) Find a basis for the null space $Null(A)$ of A.

b) Find a basis for the column space of A.

c) Is the vector $\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ in the column space of A? **Justify** your answer!

2. (10 points) Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane $x + 2y + z = 0$.

3. (24 points) Determine which of the following sets in $\mathbb{R}^n$ is a subspace. If it is not, find a property in the definition of a subspace which this set violates. If it is a subspace, find a matrix A such that this set is either $Null(A)$ or $Col(A)$.

(a) $\left\{ \begin{bmatrix} a-b \\ b-c \\ 2c+3d \\ c-a \end{bmatrix} \ : \ a, b, c, d \text{ are arbitrary real numbers} \right\}$

(b) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ : \ x, y, z \text{ are real numbers satisfying } x = y + 2z + 3 \right\}$

(c) $\left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \ : \ x, y, z, w \text{ are real numbers satisfying } \begin{array}{rcl} x+y & = & z+w \\ x+z & = & y+w \end{array} \right\}$

4. (16 points) a) Compute the area of the parallelogram in $\mathbb{R}^2$ with vertices $(1,1)$, $(3,4)$, $(6,2)$, $(8,5)$. **Caution**: note that $(0,0)$ is not a vertex.

b) Compute the volume of the parallelepiped in $\mathbb{R}^3$ with vertices $\vec{0}$, v_1, v_2, v_3, $v_1 + v_2$, $v_1 + v_3$, $v_2 + v_3$, $v_1 + v_2 + v_3$ where

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad v_3 = \begin{pmatrix} 0 \\ 1 \\ -4 \end{pmatrix}$$

c) Use your answer in part (b) and the algebraic properties of determinants to compute the volume of the parallelepiped obtained if v_3 is replaced by

$v_3' = av_1 + bv_2 + cv_3$ where a, b, c are real numbers. (Express your answer in terms of a, b, c).

5. a) (4 points) Let A, B, and C be 3×3 matrices satisfying the equation

$$B = ACA^{-1}$$

with A invertible. Solve for C in terms of A and B.

b) (8 points) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Compute its inverse A^{-1}.

c) (4 points) Compute the $(2,2)$ entry of C in part (a) if A is given in part (b) and $B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$

6. (10 points) Let $\mathbb{P}_2$ be the vector space of polynomials of degree ≤ 2. Recall that a vector in $\mathbb{P}_2$ is a polynomial $p(t)$ of the form $p(t) = a_0 + a_1 t + a_2 t^2$ where the coefficients a_0, a_1, a_2 are arbitrary real numbers.

   (a) Show that the subset H of $\mathbb{P}_2$ of polynomials $p(t)$ of degree ≤ 2 which in addition satisfy

   $$p(1) = 0$$

   is a *subspace* of $\mathbb{P}_2$. (The straightforward answer would include the definition of a subspace and a verification that H satisfies all the properties.)

   (b) Find a basis for H. **Explain** why the set you found is linearly independent and why it spans H.