Practice TEST 2

1. (*20 points*) Let $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and let $\vec{v}_2 \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$. Let V be the subspace spanned by $\vec{v}_1$ and $\vec{v}_2$.

a. (*5 pts*) Prove that $\vec{v}_1$ is not perpendicular to $\vec{v}_2$.
b. (*8 pts*) Find an orthonormal basis for V.
c. (*7 pts*) What is the matrix for orthogonal projection onto V?

2. (*17 points*) Find the quadratic polynomial $p(t) = a + bt + ct^2$ that best (in the least squares sense) fits the following data.

t	-1	0	1	2
y	1	1.5	2	3

3. (*28 points*) Let $V \subseteq C^\infty$ be subspace spanned by $\{e^x, xe^x, x^2e^x\}$. Let $\mathcal{B}$ be the ordered basis

$$\mathcal{B} = (e^x, xe^x, x^2e^x).$$

a. (*4 pts*) What is the dimension of V?
b. (*8 pts*) Let $D : V \to V$ be the linear transformation given by $D(f) = f'$. Express D as a matrix with respect to the basis $\mathcal{B}$. i.e. Compute $\mathrm{Mat}_{\mathcal{B}}^{\mathcal{B}}(D)$.
c. (*8 pts*) Let $A = \mathrm{Mat}_{\mathcal{B}}^{\mathcal{B}}(D)$. You can check that

$$A^3 - 3A^2 + 3A - 1 = 0.$$

Consider the function $f(x) = 2e^x - 13xe^x + \sqrt{2}x^2e^x$. What does the above tell you about

$$f''' - 3f'' + 3f' - f?$$

d. (*8 pts*) Suppose you want to find functions u such that

$$u'''(x) - 3u''(x) + 3u'(x) - u(x) = x.$$

Verify that $u(x) = -x - 3$ is a solution. Find another one.

4. (*15 points*) Find a basis for the space perpendicular to the solutions of

$$x_1 + 3x_2 - x_3 + x_4 = 0$$
$$-2x_1 + 2x_2 + x_3 + x_4 = 0$$

5. (*20 points*) Let P_5 denote the vector space of polynomials of degree at most 5. Let $S \subseteq P_5$ denote the subset of polynomials p such that

$$p''(2) = p(4).$$

Show that S is a subspace of P_5 and compute a basis of S.

Before test 2:

1. Make sure you can define the following words:
   (a) linear transformation
   (b) subspace
   (c) linearly independent
   (d) rank
   (e) kernel
   (f) image
   (g) span
   (h) dimension
   (i) similar matrices
   (j) vector space
   (k) transpose of a matrix
   (l) orthogonal matrix
   (m) symmetric matrix
   (n) skew-symmetric matrix
   (o) orthonormal basis
2. Make sure you can do *Gaussian Elimination* and *Gram-Schmidt*, and you know what each is good for.
3. Make sure you can solve a linear system.
4. Make sure you can state the *Rank-Nullity Theorem* and fully appreciate all of its consequences.