Practice Problems Math 235 Spring 2007

1. Write the system of equations as a matrix equation and find all solutions using Gauss elimination:

$$x + 2y + 4z = 0, -x + 3y + z = -5, 2x + y + 5z = 3.$$

2. What does it mean for a vector to be in the kernal of a matrix A. Let A be the matrix $\begin{pmatrix} 1 & 2 & 5 \\ -2 & 0 & -2 \\ 3 & -1 & 1 \end{pmatrix}$. Is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ an element of the kernal of A? Why?

3. Define what it means for a set s to be a basis of a subspace $V \subset \mathbb{R}^n$. Let

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ -1 & 0 & 1 & -1 \\ -1 & 4 & 3 & -5 \end{pmatrix}.$$

Give a set of vectors that span $ker(A)$ and that are independent.

4. Let A be a n by m matrix, so A gives a function from $\mathbb{R}^m$ to $\mathbb{R}^n$. Let $x_1, x_2 \in \mathbb{R}^m$. Assume that $A(x_1) = A(x_2)$. Show that $x_1 - x_2$ is in the kernal of A.

5. Let $u = (u_1, u_2)$ be a vector of length 1. Let A be a matrix whose effect on the plane is to reflect about the line through the origin and u. Let $v = (-u_2, u_1)$. In terms of u and v what is $A(u)$? what is $A(v)$? Write $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ as a linear combination of u and v. Use the answer to the previous question to compute $A(e_1)$.

6. Solve the equation

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

for $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ by find the inverse of the given matrix.

7. Compute the product AB of the two matrices A, B given below, if possible. If it is not possible say why it is not possible.

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 \\ 4 & 8 \end{pmatrix}$$

The product matrix AB gives a funtion. What is the domain and what is the range of that function?

8. Find a basis of the subspace of $\mathbb{R}^3$ defined by $3x - y + z = 0$. What is the dimension of this subspace?

9. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & 1 & 3 \\ -2 & 1 & -3 \end{pmatrix}$$

. Let $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$. Find conditions on b so that the equation $Ax = b$ can be solved.

Find a basis of the image of A.

10. Let V, W be subspaces of $\mathbb{R}^n$. Assume that $V \subset W$ and that the dimension of V is equal to the dimension of W. Show $V = W$.