TEST 2

Math 235
Fall 2006

Name: $\underbrace{\hspace{6cm}}$

Read all of the following information before starting the exam:

- Do all work to be graded in the space provided. If you need extra space, use the reverse of the page and indicate on the front that you have continued the work on the back.
- *You must support your answers with necessary work. Carl Friedrich Gauss was a pretty bright guy. Unsupported answers will receive zero credit.*
- Circle or otherwise indicate your final answers.
- You do not need to simplify expressions such as $\sqrt{11}$, e^2, π with decimal approximations.
- This test has 7 problems and is worth 100 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- If you have read of all these instructions, check this box for one bonus point $\square$.
- Good luck!

1. (*2 points*) What was Gauss's first name?

2. (*15 points*) Which of the following sets are subspaces of vector space $\mathcal{P}_5$?

Set	Subspace	Not a subspace
$\{p \in \mathcal{P}_5 \mid p'(0) > 0\}$		
$\{p \in \mathcal{P}_5 \mid p'(0) = 0\}$		
$\{p \in \mathcal{P}_5 \mid p'(4) = 0\}$		
$\{p \in \mathcal{P}_5 \mid p'(0) = 4\}$		
My other dog Pepsi		

3. (*18 points*) Suppose that

- O and N are an orthogonal matrices.
- S and T are symmetric matrices.
- E and W are skew-symmetric matrices.

Write *True* for the statements that MUST be true, and write *False* otherwise.

a. (*2 pts*) ____________________ W is a square matrix.

b. (*2 pts*) ____________________ E is an invertible matrix.

c. (*2 pts*) ____________________ S is a square matrix.

d. (*2 pts*) ____________________ T is an invertible matrix.

e. (*2 pts*) ____________________ O is a square matrix.

f. (*2 pts*) ____________________ N is an invertible matrix

g. (*2 pts*) ____________________ SON is an orthogonal matrix.

h. (*2 pts*) ____________________ $ST - TS$ is a skew-symmetric matrix.

i. (*2 pts*) ____________________ EE^t is a symmetric matrix.

4. (*14 points*) Let V be the plane in $\mathbb{R}^3$ spanned by $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 7 \\ -8 \end{pmatrix}$.

a. (*7 pts*) Find an orthonormal basis of V.

b. (*7 pts*) Find the orthogonal projection of $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ onto V.

5. (*15 points*) Let $\mathbb{C}$ denote the *complex numbers* with the basis $\mathcal{B} = (1, i)$. Let T be the linear transformations $T(z) = (3 + 4i)z$ from $\mathbb{C}$ to $\mathbb{C}$.

a. (*7 pts*) Find the matrix of T in the basis $\mathcal{B}$. i.e. Compute $A = \mathrm{Mat}_{\mathcal{B}}^{\mathcal{B}}(T)$.

b. (*8 pts*) Compute the QR factorization of A.

6. (*20 points*) Let $\mathbb{R}^{2\times 2}$ denote the 2×2 matrices. There is a map called *trace*, $\mathrm{Tr} : \mathbb{R}^{2\times 2} \to \mathbb{R}$, that is defined by

$$\mathrm{Tr}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d.$$

a. (*3 pts*) What is the dimension of $\mathbb{R}^{2\times 2}$?

b. (*6 pts*) Verify that Tr is a linear transformation.

c. (*6 pts*) A matrix A is said to be *traceless* if $\mathrm{Tr}(A) = 0$. Let S denote the subspace of traceless 2×2 matrices. What is the dimension of S? Find a basis for S.

d. (*5 pts*) Consider the linear transformation $Q : S \to \mathbb{R}^2$ defined by

$$Q(M) = M\begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Find the kernel of Q.

7. (*16 points*) Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

a. (*12 pts*) Find the least squares solutions of $A\vec{x} = \vec{b}$.

b. (*4 pts*) Find a point in the image of A that is as close to $\vec{b}$ as possible.

BONUS Survey: 1 bonus point total for completion

1. $327 \times 151 =$

2. Out of 100 points, what do you think is your score on this test?

3. Out of 100 points, what do you think is the class average on this test?