Math 235H Written homework assignment 4 Fall 2020

• **Please follow carefully the instructions on the document titled "Submitting PDF documents via Gradescope" (see the PDF file attachment to the announcement "Gradescope - our hand written homework submission platform" on our Moodle page). After you upload the PDF file to Gradescope you will need also to respond to the Gradescope prompt asking you to indicate, for each of the problems, the page in which the solution to that problem is found. If you have solutions to more than one problem in a page, make sure that the problem number is large and clear.**

1. Let $A = \begin{pmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{pmatrix}$ be a 4×4 matrix with rows $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$. Compute det $\begin{pmatrix} 6\vec{v}_1 + 2\vec{v}_4 \\ \vec{v}_2 \\ \vec{v}_3 \\ 3\vec{v}_1 + \vec{v}_4 \end{pmatrix}$, if $\det(A) = 4$.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with standard matrix A. Assume that for two non-zero vectors $\vec{v}_1$ and $\vec{v}_2$ we have $T(\vec{v}_1) = 5\vec{v}_1$ and $T(\vec{v}_2) = 7\vec{v}_2$.
   (a) Show that v_1 and v_2 must be linearly independent.
   (b) Show that $\det(A) = 35$. Hint: Let B be the matrix $(\vec{v}_1\vec{v}_2)$ with columns $\vec{v}_1$ and $\vec{v}_2$. Compute the determinant of the product $\det(AB)$ in two ways.

3. (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the reflection of the plane with respect to a line L through the origin and A the standard matrix of T. Show that $\det(A) = -1$. Hint: Use parts 5(b) and 5(c) in Homework 2 and imitate the argument you used in problem 2(b).

   (b) Compute the determinant of the standard matrix of the reflection R of $\mathbb{R}^3$ with respect to a plane given in Equation (1) of Homework 3 Problem 2 (for any $\vec{u}$).

4. Let A be an $n \times n$ matrix, I_n the $n \times n$ identity matrix, and O the $n \times n$ matrix all of which entries are zero. Show that if $A^2 + 3A + 4I_n = O$, then A is invertible. Hint: $3A = 3I_nA$. Then express A^{-1} in terms of A.

5. Consider the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & t \\ a^2 & b^2 & t^2 \end{pmatrix}$ for two distinct numbers a and b. We define the function $f(t) = \det(A)$.
   (a) Show that $f(t)$ is a quadratic polynomial in t. What is the coefficient of t^2.
   (b) Explain why $f(a) = f(b) = 0$. Conclude that $f(t) = k(t - a)(t - b)$, for some constant k. Find the constant k using your work in part 5a.
   (c) For which values of t is the matrix A invertible?