Math 235H Written homework assignment 1 Fall 2020

• **Please follow carefully the instructions on the document emailed to you titled "Submitting PDF documents via Gradescope". After you upload the PDF file to Gradescope you will need also to respond to the Gradescope prompt asking you to indicate, for each of the problems, the page in which the solution to that problem is found. If you have solutions to more than one problem in a page, make sure that the problem number is large and clear.**

Determine whether the statements 1 to 7 below are true or false, and carefully justify your answer.

1. There exists a 2×2 matrix A, such that $A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $A \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

2. There exists scalars a and b such that the columns of the matrix $\begin{pmatrix} 0 & 1 & a \\ -1 & 0 & b \\ -a & -b & 0 \end{pmatrix}$ span the whole of $\mathbb{R}^3$.

3. If a vector $\vec{u}$ is a linear combination of the vectors $\vec{v}$ and $\vec{w}$, and $\vec{v}$ is a linear combination of the vectors $\vec{p}$, $\vec{q}$, and $\vec{r}$, then $\vec{u}$ is a linear combination of the vectors $\vec{p}$, $\vec{q}$, $\vec{r}$, and $\vec{w}$.

4. If A is any 4×3 matrix, then there exists a vector $\vec{b}$ in $\mathbb{R}^4$ such that the system $A\vec{x} = \vec{b}$ is inconsistent.

5. If A is a 4×3 matrix with a pivot position in every column and $A\vec{v} = A\vec{w}$, for two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^3$, then the vectors $\vec{v}$ and $\vec{w}$ must be equal.

6. Let A and B be $n \times n$ matrices, such that the systems $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ each has only the trivial solution $\vec{x} = \vec{0}$, then A can be transformed into B by means of elementary row operations.

7. If $A = (\vec{u}\ \vec{v}\ \vec{w})$ and the row reduced echelon form of A is $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$, then the equation $\vec{w} = 3\vec{u} + 2\vec{v}$ must hold.

8. Show that if $\vec{w}$ belongs to span$\{\vec{t}, \vec{u}, \vec{v}\}$ but $\vec{w}$ does not belongs to span$\{\vec{t}, \vec{u}\}$, then $\vec{v}$ belongs to span$\{\vec{t}, \vec{u}, \vec{w}\}$.