Math 235 Midterm 2 Spring 2001

1. (24 points) You are given below the matrix A together with its row reduced echelon form B

$$A = \begin{pmatrix} 1 & 1 & 3 & 0 & 1 & 0 \\ 0 & 2 & 4 & 2 & 2 & 2 \\ 2 & 1 & 4 & -1 & 1 & 0 \\ 1 & 1 & 3 & 0 & 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Note: you do **not** have to check that A and B are indeed row equivalent.

a) Find a basis for the null space $Null(A)$ of A.

b) Find a basis for the column space of A.

c) Is the vector $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ in the column space of A? Use part b to **Justify** your answer!

2. (16 points) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$.
Compute i) A^{-1}, ii) B^{-1}, and iii) $(AB)^{-1}$. Check your answers in parts i and ii by calculating AA^{-1} and BB^{-1}.

3. (16 points) a) Let A, B and C be 4×4 matrices satisfying

$$B = ACA^{-1} + 2A,$$

with A invertible. Solve for C in terms of A and B.

b) Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{bmatrix}$. Find a matrix B satisfying $AB = C$.

c) Check your answer in part b by calculating AB.

4. (18 points) Determine which of the following sets in $\mathbb{R}^n$ is a subspace. If it is not, **find** a property in the definition of a subspace which this set violates. If it is a subspace, **find** a matrix A such that this set is either $Null(A)$ or $Col(A)$.

(a) $\left\{ \begin{bmatrix} x_1 + 3x_3 \\ 3x_2 - 2x_3 \\ 2x_3 - x_1 \\ 5x_1 + 3x_2 - x_3 \end{bmatrix} : x_1, x_2, x_3 \text{ are arbitrary real numbers} \right\}$

(b) $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_1, x_2, x_3, x_4 \text{ are real numbers satisfying } \begin{array}{rcl} x_1 + x_2 + x_3 & = & x_4 \\ 5x_2 & = & 4x_3 \end{array} \right\}$

(c) The unique plane in $\mathbb{R}^3$ passing through the three points

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

5. (16 points) a) Compute the area of the **triangle** in $\mathbb{R}^2$ with vertices $(0,0)$, $(1,2)$, $(3,1)$. *Hint*: Find a parallelogram whose area is twice that of the triangle.

   b) Compute the volume of the parallelepiped in $\mathbb{R}^3$ with vertices

   $\vec{0}$, v_1, v_2, v_3, $v_1 + v_2$, $v_1 + v_3$, $v_2 + v_3$, $v_1 + v_2 + v_3$, where

$$v_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \quad v_3 = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$$

   c) Use your answer in part (b) and the algebraic properties of determinants to compute the volume of the parallelepiped obtained if v_1, v_2, v_3 are replaced by w_1, w_2, w_3, where $w_i = 2v_i$, for $i = 1, 2, 3$.

6. (10 points) Let $\mathbb{P}_2$ be the vector space of polynomials of degree ≤ 2. Recall that a vector in $\mathbb{P}_2$ is a polynomial $p(t)$ of the form $p(t) = a_0 + a_1 t + a_2 t^2$, where the coefficients a_0, a_1, a_2 are arbitrary real numbers.

   (a) Find a polynomial $p(t)$, of degree at most 2, satisfying $p(0) = 4$, $p(1) = 1$, and $p(2) = 0$.

   (b) The subset H of $\mathbb{P}_2$, of polynomials $p(t)$ of degree ≤ 2, which in addition satisfy

$$p(2) = 0$$

   is a *subspace* of $\mathbb{P}_2$. (You may assume this fact). Find a basis for H. **Explain** why the set you found is linearly independent and why it spans H.