PRACTICE FOR MATH 132 EXAM #1, BASED ON EXAM #1, SPRING 2000

Disclaimer: Your instructor covers far more materials that we can possibly fit into a four/five questions exams. These practice test, taken verbatim from the Math 132 Spring 2000 Exam #1, are meant to give you an idea of the kind and varieties of questions that were asked within the time limit of that particular tests. In addition, the scope, length and format of these old exams might change from year to year. Users beware!

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1. In the following $\Delta x = 3/n$ and $x_i = 1 + i(\frac{3}{n})$.

(a) (7 points) Express $\lim_{n\to\infty} \sum_{n=1}^{\infty} \sqrt{1+x_i} \cdot \left(\frac{3}{n}\right)$ as a definite integral.

(b) (8 points) Using the Fundamental Theorem of Calculus, evaluate the limit in (a) above. You must show how you obtain the anti-derivative leading to your answer. No credit will be given for a calculator result.

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2. (10 points) The function $r(t)$ is the rate of change of the U.S. population in units of people per year where time, t, is measured in years, and $t = 0$ corresponds to the beginning of 1950. $P(t)$, the population at time t, is assumed to have a continuous derivative.

What is the interpretation of $\int_0^{25} r(t)dt$? Explain your **reasoning** in a sentence.

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3. Georgia Instrument Company has set up a production line to manufacture a new calculator. The rate of production of these calculators after t weeks is

$$4000(1 + te^{-t}) \text{ calculators/week.}$$

Time $t = 0$ coresponds to the beginning of production when the factory opens.

(a) (10 points) Calculate $P(t)$, the total number of calculators produced during the first t weeks of production.

(b) (5 points) How many calculators are produced during the $4th$ week? **NOTE:** the time period $(0, 1]$ is the first week.) Express the answer first as a definite integral and use the result in part (a).

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4. The function $s(t)$ is the position on the x-axis of a particle at time t. The velocity is denoted by $v(t)$ and the acceleration by $a(t)$. Suppose that $a(t) = 2t+4, v(0) = -5, s(0) = 0$ and $0 \le t \le 5$. Calculate

(a) (5 points) $v(t)$

(b) (5 points) $s(t)$

(c) (5 points) The **displacement** over the time interval $[0, 5]$.

(d) (5 points) The **distance travelled** by the particle over the time interval $[0, 5]$.

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5. **Use the Fundamental Theorem of Calculus** to compute the area of the triangle in the XY plane with vertices at $(0, 5), (2, -2), (5, 1)$.

As a first step, draw a picture on the XY axes, label the vertices, and put the equations of the line segments forming the triangle next to the segments.

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6. A solid figure has as its base the region in the first quadrant trapped inside the ellipse $\frac{x^2}{L^2} + 4y^2 = 1$ where $L > 0$ is a constant (the 'length' of this solid).

Cross sections of this solid perpendicular to the x-axis are rectangles with width $w(x)$ and height $h(x) = 3x$.

(a) (8 points) Set up a definite integral to calculate the volume of this solid figure.

(b) (7 points) Evaluate this integral and express the volume in terms of the constant L. Show how you found the anti-derivative.

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7. (10 points) A spring has a natural length of 20 inches. A force of 50 lbs is required to keep it stretched to a length of 25 inches.

Calculate in <u>**foot-pounds**</u> the work required to stretch the spring from 20 inches to 30 inches. Assume that the spring satisfies Hooke's law.