My Sol'n

MATH 132H FALL 2012
EXAM 1 - SYMBOLIC PART

This is the **FIRST PART** of the exam. It consists of 3 questions. Do not spend more than 30 minutes on this part!

Calculators and notes are **NOT** allowed on this part.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

$$\begin{aligned}
\cos^2(\theta)+\sin^2(\theta) &= 1.\\
\sec^2(\theta)-\tan^2(\theta) &= 1.\\
\cos^2(\theta) &= \frac{1+\cos(2\theta)}{2}\\
\sin^2(\theta) &= \frac{1-\cos(2\theta)}{2}\\
\sin(2\theta) &= 2\sin(\theta)\cos(\theta)\\
\cos(2\theta) &= \cos^2(\theta)-\sin^2(\theta)
\end{aligned}$$

4 pts

assume $\sec(\theta)>0$

1 pts

1. (10) Evaluate $\int \frac{1}{x^2\sqrt{9+x^2}}dx$ $=$ $\int \frac{1}{9\tan^2(\theta)\, 3\underbrace{\sqrt{1+\tan^2(\theta)}}_{\sec(\theta)}}\, 3\sec^2(\theta)\,d\theta$

$x = 3\tan(\theta)$

$dx = 3\sec^2(\theta)\,d\theta$

$$= \frac{1}{9}\int \underbrace{\frac{\cos^{\cancel{2}}(\theta)}{\sin^2(\theta)}\,\frac{1}{\cancel{\cos(\theta)}}}_{\frac{\cos(\theta)}{\sin^2(\theta)}}\,d\theta \;=\; \frac{1}{9}\int \frac{1}{u^2}\,du =$$

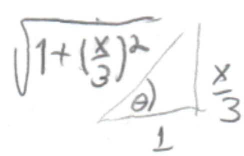
3 pts

$u = \sin(\theta)$

$du = \cos(\theta)\,d\theta$

2 pts

$$= -\frac{1}{9}u^{-1} + C = -\frac{1}{9}\frac{1}{\sin(\theta)} + C = -\frac{1}{9}\frac{\sqrt{1+\left(\frac{x}{3}\right)^2}}{\left(\frac{x}{3}\right)} + C$$



$$= \frac{-\sqrt{9+x^2}}{9x} + C$$

2. (10) Evaluate $\int \underbrace{\sin^2(x)}_{\frac{1-\cos(2x)}{2}} \underbrace{\cos^2(x)}_{\frac{1+\cos(2x)}{2}} dx$ 4 pts $= \frac{1}{4}\int 1 - \underbrace{\cos^2(2x)}_{\frac{1-\cos(4x)}{2}} dX =$

4 pts

$$= \frac{1}{4}\int \frac{1}{2} - \frac{\cos(4X)}{2}\, dX = \frac{X}{8} - \frac{\sin(4X)}{32} + C_1 =$$

2 pts

3. (10) Evaluate $\underset{I}{=}\int \underset{u}{e^{2x}} \underset{v'}{\cos(x)}dx$ 2 pts $= uV - \int u'v\, dx =$

$u' = 2e^{2x}$ $v = \sin(x)$

2 pts

$$= e^{2x}\sin(x) - 2\int \underset{u}{e^{2x}} \underset{v'}{\sin(x)}\, dX =$$

$u' = 2e^{2x}$ $v = -\cos(x)$

2 pts

$$= e^{2x}\sin(x) - 2\left\{-e^{2x}\cos(x) + \int 2e^{2x}(+\cos(x))\, dX\right\}$$

$$= e^{2x}\sin(x) + 2e^{2x}\cos(x) - 4I + C_1$$

4 pts

$$5I = e^{2x}\sin(x) + 2e^{2x}\cos(x) + C$$

$$I = \frac{1}{5}\left[e^{2x}\sin(x) + 2e^{2x}\cos(x)\right] + C_1$$

4 (12) Determine the following derivatives. Briefly justify each answer.

a) $\frac{\partial}{\partial x}\int_0^{\sin(x)} \frac{1}{1+t^2}dt.$ $= \frac{1}{1+u^2} \cdot \frac{\partial u}{\partial x} = \frac{1}{1+(\sin(x))^2} \cdot \cos(x)$

6 pts

FTC + Chain Rule $\{ u = \sin(x) \}$

6 pts

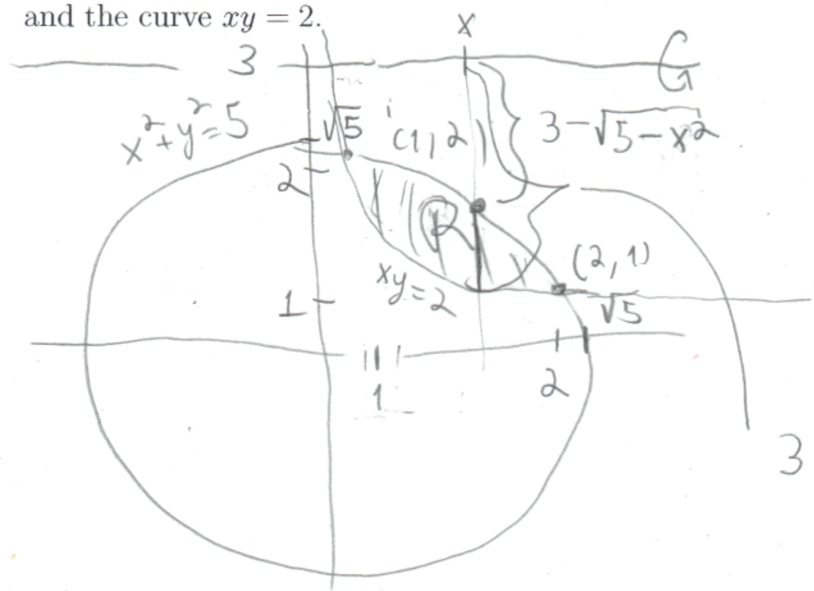
b) $\frac{\partial}{\partial x}\int_x^{e^x} \ln(t)dt$ $= \frac{\partial}{\partial x}\left[\underbrace{\int_x^1 \ln(t)dt}_{= -\int_1^x \ln(t)dt} + \int_1^{e^x} \ln(t)\,dt \right] =$

$= -\ln(x) + \underbrace{\ln(e^x)}_{x} \cdot e^x = -\ln(x) + xe^x.$

FTC + chain Rule

6 pts

5 (15) a) Sketch the region $\mathcal{R}$ in the first quadrant bounded by the curve $x^2 + y^2 = 5$, and the curve $xy = 2$.



$$x^2 + \left(\frac{2}{x}\right)^2 = 5$$

$$x^4 - 5x^2 + 4 = 0$$

$$(x^2-1)(x^2-4) = 0$$

$$x = \pm 1, \quad x = \pm 2$$

$$3 - \left(\frac{2}{x}\right)$$

9 pts

b) Set up a definite integral for the the volume of the solid obtained by revolving about the horizontal line $y = 3$ the region $\mathcal{R}$ in part a). Do **NOT** evaluate the integral.

$$\text{Vol} = \int_1^2 \pi \left(R^2(x) - r^2(x)\right) dx =$$

$$R^2(x) = \left(3 - \left(\frac{2}{x}\right)\right)^2 \qquad r^2(x) = \left[3 - \sqrt{5-x^2}\right]^2$$

$$\pi \int_1^2 \left(3 - \left(\frac{2}{x}\right)\right)^2 - \left(3 - \sqrt{5 - x^2}\right)^2 dx$$

7 pts

6 (15) a) Express the integral $\int_1^5 \frac{1}{1+x^2}dx$ as a limit, as n goes to infinity, of Riemann sums with n sub-intervals of equal length, using right endpoints as sample points.

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}$$

$x_0 = 1 \quad x_1 = 1+\frac{4}{n} \quad x_{i-1} \quad x_i = 1+\frac{4i}{n} \quad 5 = x_n$

$1+\frac{4i}{n}$ = right endpoint of i-th interval $[x_{i-1}, x_i]$.

$$\int_1^5 \frac{1}{1+x^2}dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\underbrace{\Delta x}_{\frac{4}{n}} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+\left(1+\frac{4i}{n}\right)^2} \cdot \frac{4}{n}$$

8 pts

b) Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\left(\frac{2i}{n}\right)}_{x_i} e^{[(2i/n)^2]} \underbrace{\left(\frac{2}{n}\right)}_{\Delta x}$ as a definite integral and use it to evaluate the limit.

$x_1 = \frac{2}{n}, \ldots, x_n = 2$.

$a = 0, \quad b = 2$

$$\lim_{n\to\infty} R.S = \int_0^2 x e^{\underbrace{(x^2)}_{u}} dx = \frac{1}{2}\int_0^4 e^u \, du = \frac{e^4 - e^0}{2} = \frac{e^4-1}{2},$$

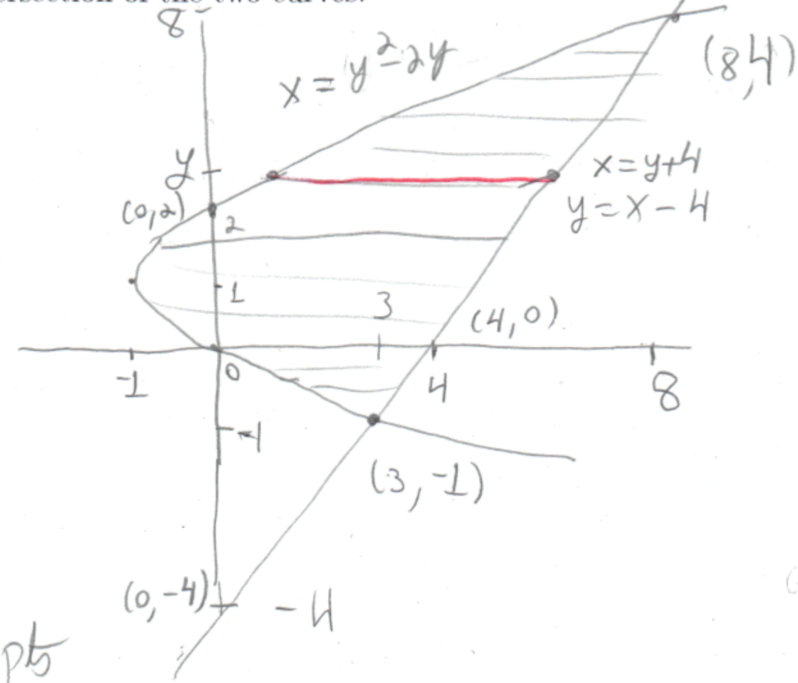
$u = x^2$

7 pts

$= (y-1)^2 - 1$

7 (15) a) Sketch the region in the plane bounded by the curves $x = y^2 - 2y$ and $y = x - 4$. Label the coordinates of all the x and y intercepts and the points of intersection of the two curves.

$x = y + 4$



$y+4 = y^2-2y$

$y^2 - 3y - 4 = 0$

$(y+1)(y-4) = 0$

$y = -1$, or $y = 4$

$x = 3$, or $x = 8$

8 pts

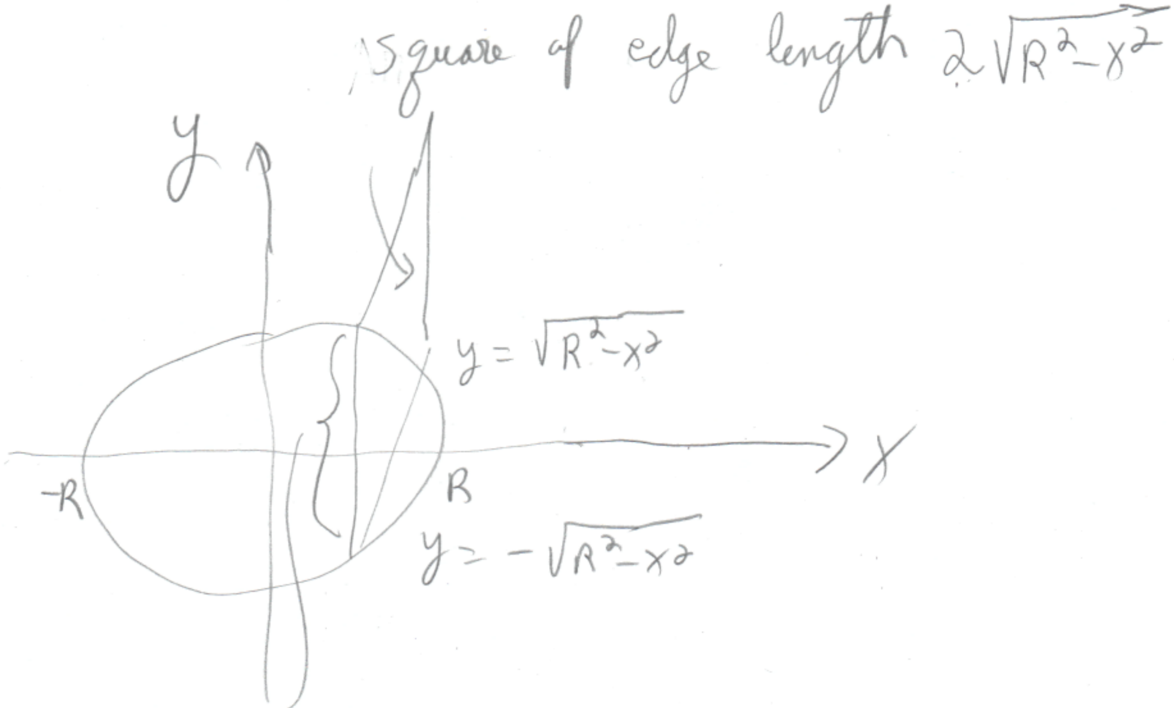
b) Find the area of the region in part a.

$$\int_{-1}^{4} \Big[\underbrace{(y+4)}_{f(y)} - \underbrace{(y^2-2y)}_{g(y)}\Big]\,dy = \int_{-1}^{4} -y^2+3y+4\,dy =$$

$$= \left[-\frac{y^3}{3} + \frac{3}{2}y^2 + 4y\right]_{-1}^{4} = \left(-\frac{\overset{64}{\overbrace{(4)^3}}}{3} + 24 + 16\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right)$$

$$= -\frac{65}{3} + \overbrace{42\tfrac{1}{2}}^{85/2} = \frac{125}{6}$$

8 (13) Find the volume of the solid S, whose base is a circular disk of radius R. Parallel cross-sections perpendicular to the base are squares. Carefully justify your answer.



$$\int_{-R}^{R} \underbrace{A(x)}_{=\left(2\sqrt{R^2-x^2}\right)^2} \, dx = 4\int_{-R}^{R} R^2-x^2 \, dx =$$

$$= 4\left[R^2x - \frac{x^3}{3}\right]_{-R}^{R} = 4\left[\underbrace{\left(R^3 - \frac{R^3}{3}\right)}_{\frac{2R^3}{3}} - \left(-R^3 + \frac{R^3}{3}\right)\right]$$

$$= \boxed{\frac{16R^3}{3}}$$