Problem 3.3.5 page 143:

Let X be an m-dim'l cpt Kähler manifold, $Y \overset{i}{\hookrightarrow} X$ smooth hypersurface, such that $\omega := P.D.[Y] \in H^2(X,\mathbb{R})$ is a Kähler class. Show that $i^* : H^k(X,\mathbb{R}) \to H^k(Y,\mathbb{R})$ is injective for $k \leq m-1$.

Proof: Let $0 \neq \alpha \in H^k(X,\mathbb{R})$ and assume that $k \leq m-1$. By Hard Lefschetz Theorem, $\alpha \cup \omega^{m-k}$ is a non-zero class in $H^{2m-k}(X,\mathbb{R})$. Poincare Duality implies that there exists a class $\beta \in H^k(X,\mathbb{R})$, such that
$$\int_X \alpha \cup \omega^{m-k} \cup \beta \neq 0.$$
Now $\omega = PD[Y]$, so

$$0 \neq \int_X \alpha \cup \underbrace{\omega^{m-k}}_{\underbrace{\omega}_{= P.D.[Y]} \cup\, \omega^{\overbrace{m-k-1}^{\geq 0}}} \cup \beta = \int_Y i^*(\alpha) \cup i^*(\omega^{m-k-1} \cup \beta).$$

Hence, $i^*(\alpha) \neq 0$. We conclude that i^* is injective. $\square$