1. Let X, Y be compact Riemann surfaces, $F : X \to Y$ a non-constant holomorphic map, and $D \in \mathrm{Div}(Y)$ a divisor. Let Q be the linear subsystem, of the complete linear system $|F^*D|$, corresponding to the linear subspace $\mathbb{P}(F^*L(D)) \subset \mathbb{P}(L(F^*D))$. Assume, that $|D|$ is base-point-free. Show that the linear system Q is base-point-free, and the associated morphism ϕ_Q is the composition
$$\phi_Q \quad = \quad \phi_D \circ F.$$

2. Let X be the hyperelliptic curve of genus g compactifying the affine plane curve given by $v^2 = h(u)$, where h is a polynomial of degree $2g+1$ or $2g+2$ with distinct roots. Denote by $\pi : X \to \mathbb{P}^1$ the degree 2 map. Show that the holomorphic map $\phi := [1 : u : ... : u^{g-1}]$ is the composition of π and the Veronese embedding of $\mathbb{P}^1$ as the rational normal curve of degree $g-1$ in $\mathbb{P}^{g-1}$.

3. Set $D := (g-1)\infty \in \mathrm{Div}(\mathbb{P}^1)$. Show that $K := \pi^*D$ is a canonical divisor of X.

4. Use the extra problem for homework 6 to show that $\dim(L(K)) = g$. Conclude, that the holomorphic map $\phi := [1 : u : ... : u^{g-1}]$ is the the morphism ϕ_K associated to the complete canonical linear system $|K|$.