Math 621 Homework Assignment 8 Spring 2013

Due: Friday, April 19

1. Lang page 189 Problem 29: Let U be a connected open set, and let D be an open disk whose closure is contained in U. Let f be analytic on U and not constant. Assume that the absolute value $|f|$ is constant on the boundary of D. Prove that f has at least one zero in D. *Hint: Consider* $g(z) := f(z) - f(z_0)$ *with* $z_0 \in D$.

2. Basic Exam January 2000 Problem 9: Prove that the equation

$$20\frac{z^3}{z^2+4} = e^z$$

has 3 roots in the unit disk $\{|z| < 1\}$.

3. Basic Exam, August 99 Problem 4: Let λ be a real number larger that 1. Show that the equation $\lambda - z - e^{-z} = 0$ has exactly one solution in the half plane $\{z : \mathrm{Re}(z) > 0\}$. Moreover, the solution is real.

4. Basic Exam, January 99 problem 3:
   (a) Determine the number of zeroes of $z^5 - 2z^2 + z + 1$ in the disk $\{z : \ |z| < 10\}$.
   (b) Compute the integral $\displaystyle\int_{\{z:|z|=10\}} \frac{3z^4+1}{z^5 - 2z^2 + z + 1} dz$.

5. Compute the following integrals:
   (a) $\displaystyle\int_0^{\pi/2} \frac{dx}{a + \sin^2(x)}, \quad |a| > 1,$
   (b) $\displaystyle\int_0^{\infty} \frac{x^2 dx}{x^4 + 5x^2 + 6},$
   (c) $\displaystyle\int_0^{\infty} \frac{\cos(x)}{x^2+a^2} dx, \quad a > 0$ real,
   (d) $\displaystyle\int_0^{\infty} \frac{x\sin(x)}{x^2+a^2} dx, \quad a \geq 0$, real,
   (e) $\displaystyle\int_0^{\infty} \frac{x^{1/3}}{1+x^2} dx$