1. Consider a function \( f \in L^1([a,b]) \) and let us extend the function \( f \) to be 0 outside of \([a,b]\). For \( h > 0 \) define

\[
f_h(x) = \frac{1}{2h} \int_{x-h}^{x+h} f(t) \, dt
\]

(a) Show that \( f \) is a continuous function.

(b) Show that \( \|f_h\|_{L^1([a,b])} \leq \|f\|_{L^1([a,b])} \) for any \( h > 0 \)

(c) Show that \( \lim_{h \to 0} \|f_h - f\|_{L^1([a,b])} = 0 \).

*Hint:* Fubini.

2. Problem 7, page 147

3. Show that if a function is of absolutely continuous on \([a,b]\) then it is of bounded variation on \([a,b]\).

4. Problem 10, page 147

5. Problem 13, p. 147


*Hint:* For (a) use corollary 3.7. For (b) write \( F' = g + h \) where \( g \) is a step function and \( \int |g| \, dx \leq \epsilon \). Consider then \( F = G + H \) where \( G = \int_a^x g(t) \, dt \) and \( H = \int_a^1 h(t) \, dt \).

7. (a) Show that the function \( f \) given by \( f(0) = 0 \) and \( f(x) = x^a \sin(x^{-b}) \) for \( x \in (0,1] \) with \( a, b > 0 \) is absolutely continuous iff \( a > b \).

(b) Consider the function \( f \) given by \( f(0) = 0 \) and \( f(x) = x^2 |\sin(1/x)| \) for \( x \in (0,1] \) and \( g(x) = \sqrt{x} \). Show that \( f \) and \( g \) and \( f \circ g \) are absolutely continuous but that \( g \circ f \) is not absolutely continuous.

8. Compute the positive and negative variation of \( f(x) = x^3 - |x|, -1 \leq x \leq 1 \) and \( f(x) = \cos(x) \) for \( 0 \leq x \leq 2\pi \).

9. Problem 24, page 150

10. Problem 19, page 148