

## Homework 2

**Exercise 1:** A coin is tossed three times. What is the probability that two heads occur given that

- The first outcome was a head.
- The first outcome was a tail.
- The first two outcomes were heads.
- The first two outcomes were tails.
- The first outcome was a head and the third was a head.

**Exercise 2:** Imagine a game where a player is handed two cards (say at the beginning of Texas Hold' Em Poker). A particularly lousy opponent actually reveals one of his two cards to you and it is an ace. Consider the following cases:

1. His first card is an ace. What is the probability he has two aces? That is compute

$$P( 2 \text{ aces} \mid \text{first card is ace} )$$

2. One of his card is an ace. What is the probability he has two aces? That is compute

$$P( 2 \text{ aces} \mid \text{one card is an ace} )$$

3. One of his card is the ace of spade. hat is the probability he has two aces? That is compute

$$P( 2 \text{ aces} \mid \text{one card is the ace of spade} )$$

that his second card is an ace.

4. Are you surprised by the results?

**Exercise 3:** Suppose Math 478 has two section. In section I there is 12 female and 18 male students. In section II there are 20 female and 15 male students. The professor picks a section at random and then picks a student at random in that section. Compute

1. Probability that the student chosen is a female.
2. The conditional probability that the student is in section *II* given that she is a female

**Exercise 4:** Suppose your hand at blackjack is a total of 18 and the card up for the dealer is a 10. Using the  $\infty$  many deck assumption compute that probability that there is a tie (i.e., the dealer gets a 18 as well).

**Exercise 5:** Consider a deck consisting of the thirteen diamonds cards which we order from ace (the lowest) to king (the highest). Three players are each given one of these 13 cards.

1. Suppose the first player has the card  $x$ . Compute the probability that he has the highest card among the three players if  $x = 8, 9, 10$ .
2. Suppose the second player has the card  $y$ . She now asks the first player whether she thinks that she has the highest cards among the three. The first player, being a math major, will say YES if the probability that she has the highest card exceeds  $\frac{1}{2}$  and NO if it is below  $\frac{1}{2}$ . If the first player says NO, what is the probability that the second player has the highest card among the three players if  $y = 8, 9, 10$ .
3. Suppose the third player has the card  $z$ . The third player which heard the answer of the first player to the second player now asks the second player if she thinks she had the highest card among the three. If the second player says NO, compute the probability that the second player has the highest card among the three players if  $z = 8, 9, 10$ .

**Exercise 6: False positives, Sensitivity and Specificity.** Bayes formula is very much used in epidemiology. Suppose we deal with a disease and we have test for the disease. We know

- The *sensitivity* of the test is 99%, this means that if you have the disease, the test is positive with probability 0.99.
- The *specificity* of the test is 98%, this means that if you do not have the disease, the test is negative with probability 0.98.
- The *prevalence* of the disease is one in two hundred, this means that the probability to carry the disease is 0.005.

The probability that someone with a positive test actually has the disease is called the *positive predictive value* of the test. The probability that someone with a negative test actually does not have the disease is called the *negative predictive value* of the test.

Express the positive and negative predictive value of the test using conditional probabilities and compute them using Bayes formula.

**Exercise 7: Monty's Hall** In this problem you will analyze how important the assumptions are in Monty's Hall problem.

1. Assume as before that the prize is randomly put behind one the three doors. However after watching the game many times you notice that the show host who is usually standing near door 1 is lazy and he tends to open the door closest to him. For example if you pick door 1, he opens door 2 75% of the time and door 3 only 25% of the time. Does this modify your probability of winning if you switch (or not switch)?
2. Suppose you have been watching Monty's Hall game for a very long time and have observed that the prize is behind door 1 45% of the time, behind door 2 40% of the time and behind door 3 15% of the time. The rest is as before and you assume that the host opens door at random (if he has a choice).
  - (a) When playing on the show you pick door 1 again and the host opens one empty door. Should you switch? Compute the various cases.
  - (b) If you know you are going to be offered a switch would it be better to pick another door? Explain.