

DEPARTMENT OF MATHEMATICS AND STATISTICS  
UNIVERSITY OF MASSACHUSETTS  
MATH 131 Fall 2003  
FINAL EXAM

Your Section Number: \_\_\_\_\_

Your Instructor's Name: \_\_\_\_\_

Print Your Name: \_\_\_\_\_

Sign Your Name: \_\_\_\_\_

This exam consists of 7 questions, some having unrelated parts. It has 8 numbered pages, where the last is a blank page for scratchwork.

On this exam, you may use a calculator and a page of your own notes, but no books.

It is not sufficient to just write the answers. You must show how you arrive at your answers unless instructed otherwise. If you draw a graph, show the numerical scale on each axis.

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Leave the space below empty!

1. (10) \_\_\_\_\_

2. (15) \_\_\_\_\_

3. (15) \_\_\_\_\_

4. (15) \_\_\_\_\_

5. (15) \_\_\_\_\_

6. (15) \_\_\_\_\_

7. (15) \_\_\_\_\_

TOTAL (100)

1. a) (5 points)

Use **implicit differentiation** to find  $dy/dx$  when  $x^2y + xy^3 = 6$ .

b) (5 points) Find an equation for the tangent line to this curve at the point  $(2, 1)$ .

2. (15 points) Use l'Hospital's Rule and other limit rules (showing steps) to determine:

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{x^3}{e^x - 1}$$

$$\lim_{x \rightarrow 0^+} x \ln x$$

3. (5 points) Use the **definition of the derivative as a limit** (with  $h \rightarrow 0$ ) to obtain the standard formula  $f'(x) = -1/x^2$  for the derivative at  $x$  of the function  $f(x) = \frac{1}{x}$ .

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(10 points) Let  $f(x) = \frac{x^2 + 1}{x^2 - 4}$ . Determine all vertical asymptotes  $x = a$ , computing in each case  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$ .

Determine all horizontal asymptotes  $y = L$ .

4. (5 points) Use **logarithmic differentiation** to find  $dy/dx$  when  $y = x^{\ln x}$ .

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(10 points) Suppose  $f$  is defined for  $x \neq \pm 1$ , with  $f'(x) = \frac{-2x}{(x^2 - 1)^2}$  and  $f''(x) = \frac{6x^2 + 2}{(x^2 - 1)^3}$ . Without knowing  $f$ , determine:

Intervals on which  $f$  is increasing?

Intervals on which  $f$  is decreasing?

Local maxima of  $f$  occur at  $x =$

Local minima of  $f$  occur at  $x =$

Intervals on which the graph of  $f$  is concave upward?

Intervals on which the graph of  $f$  is concave downward?

Inflection points occur for  $x =$

5. (15 points) Let  $f(x) = x^5 - 5x^4 + 1$ . Determine the following (without details):

$$f'(x) =$$

Critical numbers of  $f$ ?

Intervals on which  $f$  is increasing?

Intervals on which  $f$  is decreasing?

Local maxima of  $f$  occur at  $x =$

Local minima of  $f$  occur at  $x =$

$$f''(x) =$$

Intervals on which the graph of  $f$  is concave upward?

Intervals on which the graph of  $f$  is concave downward?

Inflection points occur for  $x =$

6. Let  $f(x) = xe^x$ .

(4 points) Compute, showing steps:

$$f'(x) =$$

$$f''(x) =$$

(3 points) Find all local maxima and minima, showing use of First or Second Derivative Test.

(3 points) Find all inflection points  $(x, y)$ . **Justify!**

(5 points) Sketch the graph of  $f$ , using a suitable scale, labelling absolute maxima/minima and inflection points. (Check your answer with a graphing calculator.)

7. (15 points) Find the points  $(x, y)$  on the curve  $\sqrt{x} - y + 1 = 0$  which are closest to the point  $(1, 1)$ .

It is enough to minimize the *squared distance*  $(x - a)^2 + (y - b)^2$  between  $(1, 1)$  and a point  $(x, y)$  on the curve, written as a function  $D(x)$  of  $x$ .

**Show steps and explain why your answer gives the absolute minimum, not just a relative minimum.**



