UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **Dec. 12th, 2023**
1:00-3:00 p.m.

Your Name (Last, First)____

Student ID Number____________________

Signature______________________________ **Section Number** _____

Section	Instructor	Class Time		Section	Instructor	Class Time
1	Richard Buckman	MWF 12:20-1:10pm		9	Oussami Landoulsi	MW 4:00-5:15pm
2	Eric Heinzman	MWF 11:15-12:05pm		10	Jie Min	TuTh 10:00-11:15am
3	Richard Buckman	MWF 1:25-2:15pm		11	Jie Min	TuTh 8:30-9:45am
4	Oussami Landoulsi	MW 2:30-3:45pm		12	Jin-Cheng Guu	TuTh 2:30-3:45pm
5	Jinguo Lian	MWF 9:05-9:55am		13	Catherine Benincasa	MW 2:30-3:45pm
6	Jinguo Lian	MWF 10:10-11:00am		15	Seong Eun Jung	TuTh 2:30-3:45pm
7	Eric Heinzman	MWF 10:10-11:00am		17	Jin-Cheng Guu	TuTh 4:00-5:15pm
8	Ning Jiang	TuTh 10:10-11:15am		18	Dean Katsaros	TuThu 10:00-11:15am
				19	Connor Kennedy	TuTh 2:30-3:45pm

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- The above applies until you have submitted your exam to us and signed the attendance sheet. Do not use a cell phone or talk while waiting in line, and please wait until you exit the building to discuss anything, both for the benefit of others still taking the exam, and to prevent unintentionally spoiling the exam.
- There are thirteen (13) questions (see the following question table) and 15 pages (including the last blank page). Please check if you have consecutive page number from 1 to 15 and all listed questions, if not, please raise your hands let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (**Answers given without proper justification may receive no credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT		QUESTION	PER CENT
1(a)	8		4(a)	8
1(b)	8		4(b)	8
2(a)	6		5(a)	8
2(b)	6		5(b)	8
2(c)	6		6(a)	8
3(a)	8		6(b)	8
3(b)	8		Free	2
TOTAL QUESTIONS	13		TOTAL SCORES	100

#1. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(1a) (8 points) For each question (I)–(II), there is only one correct response, please **circle** it out.

(I) (4 points) Given $f(x) = \ln\left(\frac{(2x+1)^3}{(3x-1)^5}\right)$, Which of the following is the $\frac{df}{dx}$?.

[A] $\frac{df}{dx} = \frac{6}{2x+1} - \frac{15}{3x-1}$

[B] $\frac{df}{dx} = \frac{3}{2x+1} - \frac{5}{3x-1}$

[C] $\frac{df}{dx} = \frac{2}{2x+1} - \frac{3}{3x-1}$

[D] $\frac{df}{dx} = \frac{3(2x+1)^2(3x-1)^5 - 5(2x+1)^3(3x-1)^4}{(3x-1)^{10}}$

(II) (4 points) Given $f(x) = \frac{x^2-1}{x^2-x}$, which of the following statements is true?

[A] $f(x)$ is continuous at $x = 1$.

[B] $\lim_{x\to 1} f(x)$ does not exists.

[C] $\lim_{x\to 1} f(x) = 2$

[D] $x = 1$ is a vertical asymptote of the curve $y = f(x)$.

(1b) (8 points) Let $f(x) = 3x^2+5x-7$. Use the **limit definition of the derivative** to find $f'(x)$. (e.g. Do not use the power rule, etc.)

#2. (18 points) Evaluate the following limits:

(2a) (6 points) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$.

(2b) (6 points) $\lim_{x\to 0} \frac{e^{3x} - 1 - 3x}{x^2}$.

(2c) (6 points) $\lim_{x\to\infty}(1+\frac{5}{x})^x$.

#3. (16 points) Let $f(x) = x^3 - \frac{3}{2}x^2 - 18x + 100$. We know that f is defined for all real numbers and that: $f'(x) = 3x^2 - 3x - 18$, $f''(x) = 6x - 3$.

(3a) (8 points) Find the interval(s) on which f is increasing. Find the interval(s) on which f is decreasing. Determine the x-coordinates of all local maxima and local minima.

(3b) (8 points) Using the same function, find the interval(s) where the function is concave up. Find the interval(s) where the function is concave down. Determine the x-coordinates of all points of inflection of the graph of f(x).

(For convenience, $f(x) = x^3 - \frac{3}{2}x^2 - 18x + 100$, with $f'(x) = 3x^2 - 3x - 18$, and $f''(x) = 6x - 3$.)

#4. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(4a) (8 points) A box with a square base and open top must have a volume of 500 cm^3. Find the dimensions of the box that minimize the amount of material used.

(4b) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = x^3 + 3x^2 - 9x + 11$ on the interval [-5, 2].

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(5a) (8 points) A particle is moving with the acceleration given by: $a(t) = e^t + 2t + 2$. Given that $v(0) = 0$ and $s(0) = 0$, find the position of the particle, $s(t)$.

(5b) (8 points) Since the function $f(x) = x^2$ is continuous on the interval $[0, 10]$, and differentiable on the interval $(0, 10)$, the Mean Value Theorem applies, showing the existence of c in the interval $(0, 10)$ with certain properties. State the equation which holds and find a value of c demonstrating it.

#6. (16 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(6a) (8 points) Given a function $f(x) = 1+x^2$. Write a definite integral representing the exact area of the region under the curve $y = f(x)$ on the interval [0, 1]. Evaluate this integral using the definition of the definite integral as a limit of Riemann sums. Do not use the fundamental theorem of calculus here. You can use the following identities for sums of powers of consecutive positive integers.

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(6b) (8 points) Evaluate the Riemann sum for $f(x) = 1 + x + x^2$, $-2 \le x \le 2$ with four subintervals, taking the sample points to be the right endpoints.

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