

UNIVERSITY OF MASSACHUSETTS AMHERST  
DEPARTMENT OF MATHEMATICS AND STATISTICS

**Math 131**

**Exam 2**

**Nov. 8th, 2023  
7:00-9:00 p.m.**

**Your Name (Last, First)** \_\_\_\_\_

**Student ID Number** \_\_\_\_\_

**Signature** \_\_\_\_\_ **Section Number** \_\_\_\_\_

Section	Instructor	Class Time	Section	Instructor	Class Time
1	Richard Buckman	MWF 12:20-1:10pm	9	Oussami Landoulsi	MW 4:00-5:15pm
2	Eric Heinzman	MWF 11:15-12:05pm	10	Jie Min	TuTh 10:00-11:15am
3	Richard Buckman	MWF 1:25-2:15pm	11	Jie Min	TuTh 8:30-9:45am
4	Oussami Landoulsi	MW 2:30-3:45pm	12	Jin-Cheng Guu	TuTh 2:30-3:45pm
5	Jinguo Lian	MWF 9:05-9:55am	13	Catherine Benincasa	MW 2:30-3:45pm
6	Jinguo Lian	MWF 10:10-11:00am	15	Seong Eun Jung	TuTh 2:30-3:45pm
7	Eric Heinzman	MWF 10:10-11:00am	17	Jin-Cheng Guu	TuTh 4:00-5:15pm
8	Ning Jiang	TuTh 10:10-11:15am	18	Dean Katsaros	TuThu 10:00-11:15am
			19	Connor Kennedy	TuTh 2:30-3:45pm

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- The above applies until you have submitted your exam to us and signed the attendance sheet. Do not use a cell phone or talk while waiting in line, and please wait until you exit the building to discuss anything, both for the benefit of others still taking the exam, and to prevent unintentionally spoiling the exam.
- There are twelve (12) questions (see the following question table) and 14 pages. Please check if you have consecutive page number from 1 to 14 and all listed questions, if not, please raise your hands let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (**Answers given without proper justification may receive no credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	QUESTION	PER CENT
1(a)	8	4(b)	9
1(b)	8	5(a)	8
2(a)	8	5(b)	8
2(b)	8	6(a)	9
3(a)	8	6(b)	9
3(b)	8		
4(a)	9		
TOTAL QUESTIONS	12	TOTAL SCORES	100

#1. (16 points) Find the derivatives of the following functions. You do NOT need to simplify your answer. These two problems are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to solve part (b).

(1a) (8 points)  $f(x) = \arcsin(2x) + \frac{x^2}{x+1} + \log_5(3x+1) + \csc(x^2)$ .

(1b) (8 points)  $g(x) = \tan(x) \cos(x) + \ln(\sqrt{x}) + \cot(\sin(x)) + \sec(e^{-x})$

#2. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(2a) (8 points) Let:  $\frac{x}{y} = e^{xy}$ . Use implicit differentiation to find  $\frac{dy}{dx}$ . Your answer may be an expression involving  $x$  and  $y$ .

(2b) (8 points) Let:  $y = (2x + 3)^{x^2}$ . Use logarithmic differentiation to find  $\frac{dy}{dx}$ . You don't have to simplify your final answer, but it should be a function of  $x$  only.

#3. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know the answer to part (a), you should still attempt to find answer to part (b). Please be sure to justify your answers.

(3a) (8 points) A bacterial culture initially contains 200 cells and grows at a rate proportional to its size. After an hour the population has increased to 600. Find an expression for the number of bacteria after  $t$  hours. Your answer may involve logarithms and the number  $e$ .

(3b) (8 points) A hospital prepares a 100mg supply of technetium-99 which has a half-life of 6 hours. Find an expression for the mass of technetium-99 remaining (in mg) after  $t$  hours. Your answer may involve logarithms and the number  $e$ .

#4. (18 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(4a) (9 points) A ladder 13 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/s, how fast (in rad/s) is the angle (in radians) between the ladder and the ground changing when the bottom of the ladder is 12 ft from the wall? (That is, find the angle's rate of change when the bottom of the ladder is 12 ft from the wall.)



(4b) (9 points) If a snowball melts so that its surface area ( $S = 4\pi r^2$ ) decreases at a rate of  $5 \text{ cm}^2/\text{min}$ , find the rate (in  $\text{cm}/\text{min}$ ) at which the diameter decreases when the diameter is 10 cm. Your answer may involve the number  $\pi$ .

#5. (16 points) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(5a) (8 points) Find the linear approximation of the function  $f(x) = \sqrt{x}$  at  $x = 1$ . Use it to approximate the number  $\sqrt{1.02}$

(5b) (8 points) Compute  $\Delta y$  and  $dy$  of function  $y(x) = 2^{x^2}$  when  $x = 1$  and  $dx = \Delta x = 0.1$ . You do NOT need to simplify your answer.

#6. (18 point) Parts (a) and (b) of this problem are NOT related and can be solved independently from each other. If you don't know how to solve part (a), you should still attempt to answer part (b).

(a) (9 points) A particle moves according to a law of motion given by the displacement function  $s(t) = t^3 - 3t^2$ , where  $t \geq 0$  is measured in seconds and  $s$  is measured in feet. Find the total distance (in feet) traveled during the first 5 seconds.

(b) (9 points) If a ball is thrown vertically upward with an initial velocity of 9 ft/s, then its height after  $t$  seconds is  $s = 9t - t^2$ . (Consider up to be the positive direction.) What is the velocity (in ft/s) of the ball when it is 20 ft above the ground on its way up?

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