

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131

Exam 1

**Oct. 11th, 2023
7:00-9:00 p.m.**

Your Name (Last, First) _____

Student ID Number _____

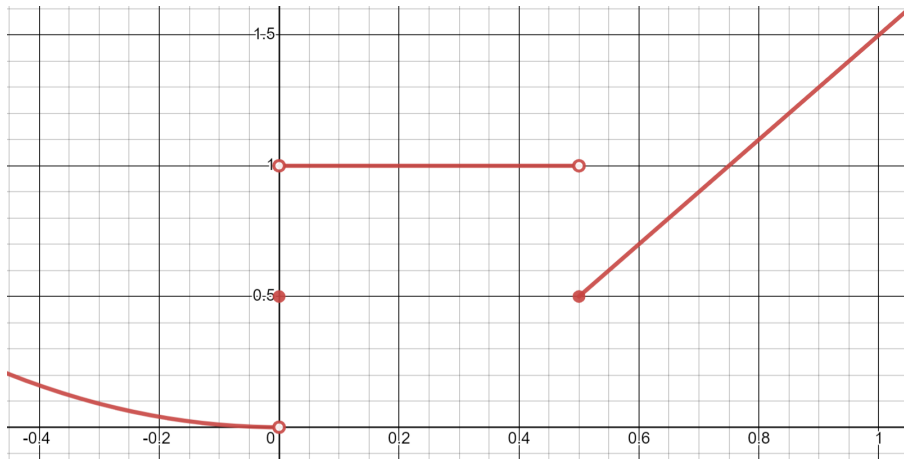
Signature _____ **Section Number** _____

Section	Instructor	Class Time	Section	Instructor	Class Time
1	Richard Buckman	MWF 12:20-1:10pm	9	Oussami Landoulsi	MW 4:00-5:15pm
2	Eric Heinzman	MWF 11:15-12:05pm	10	Jie Min	TuTh 10:00-11:15am
3	Richard Buckman	MWF 1:25-2:15pm	11	Jie Min	TuTh 8:30-9:45am
4	Oussami Landoulsi	MW 2:30-3:45pm	12	Jin-Cheng Guu	TuTh 2:30-3:45pm
5	Jinguo Lian	MWF 9:05-9:55am	13	Catherine Benincasa	MW 2:30-3:45pm
6	Jinguo Lian	MWF 10:10-11:00am	15	Seong Eun Jung	TuTh 2:30-3:45pm
7	Eric Heinzman	MWF 10:10-11:00am	17	Jin-Cheng Guu	TuTh 4:00-5:15pm
8	Ning Jiang	TuTh 10:10-11:15am	18	Dean Katsaros	TuThu 10:00-11:15am
			19	Connor Kennedy	TuTh 2:30-3:45pm

- Please turn off and put away all electronic devices (cell phones, laptops, tablets, smart watches, etc.). This is a closed book exam. No calculators, notes, or books are allowed.
- The above applies until you have submitted your exam to us and signed the attendance sheet. Do not use a cell phone or talk while waiting in line, and please wait until you exit the building to discuss anything, both for the benefit of others still taking the exam, and to prevent unintentionally spoiling the exam.
- There are twelve (12) questions (see the following question table) and 14 pages. Please check if you have consecutive page number from 1 to 14 and all listed questions, if not, please raise your hands let proctors know. Each question has its own page with extra space, so please keep your answer on the same page and side as the corresponding question. Use pencil in case you need to edit; if you need to rewrite your answer please erase it so you can keep it on the same page. Any work done elsewhere should be copied to the page if you want it to be considered.
- For each question, please provide appropriate mathematical details to justify your answer and organize your work in an unambiguous order. (**Answers given without proper justification may receive no credit.**)
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	QUESTION	PER CENT
1	16	4(b)	8
2(a)	5	5(a)	8
2(b)	6	5(b)	8
2(c)	5	6(a)	12
3(a)	8	6(b)	6
3(b)	8	Free	2
4(a)	8		
TOTAL QUESTIONS	12	TOTAL SCORES	100

#1. (16 points) Below is the graph of $f(x)$. Find the following function values or limits. If any of the quantities do not exist, clearly explain why.



(a) $f(0) =$

(b) $\lim_{x \rightarrow 0^-} f(x) =$

(c) $\lim_{x \rightarrow 0^+} f(x) =$

(d) $\lim_{x \rightarrow 0} f(x) =$

(e) $\lim_{x \rightarrow 0.5^+} f(x) =$

(f) $\lim_{x \rightarrow 0.5^-} f(x) =$

(g) $\lim_{x \rightarrow 0.5} f(x) =$

(h) $\lim_{x \rightarrow 0.2} f(f(x)) =$

#2. (16 points) Find the following limits. Your work should be presented neatly and logically. It must contain steps that mathematically justify your answer using theorems or properties covered in our Calculus course and must include enough detail to provide a logical and complete solution to the question asked in order for you to receive full credit. Using a graph or table of numbers will not receive credit as mathematical justification.

(2a) (5 points) $\lim_{x \rightarrow 1} \left(\frac{x^3 - x}{x - 1} \right)$.

(2b) (6 points) $\lim_{x \rightarrow 2} \left(\frac{\sqrt{x^2 + 6} - \sqrt{10}}{x - 2} \right)$.

(2c) (5 points) $\lim_{t \rightarrow \infty} \left(\frac{3 - e^t}{3 + 5e^t} \right)$

#3. (16 points)

(3a) (8 points) Let

$$f(x) = \begin{cases} x^2 + 2cx + 2, & \text{if } x < 2; \\ cx^3 + cx + 3, & \text{if } x \geq 2. \end{cases}$$

Use the definition of continuity and the properties of limits to determine the value of c that makes the function $f(x)$ continuous on $(-\infty, \infty)$.

(3b) (8 points) Consider the function $f(x)$ below. Use the definition of continuity and the properties of limits to (i) determine where, if anywhere $f(x)$ is discontinuous and (ii) determine where, if anywhere $f(x)$ is continuous from the left or right. Your work should be presented neatly and logically. It must contain steps that mathematically justify your answer using theorems or properties covered in our Calculus course and must include enough detail to provide a logical and complete solution to the question asked in order for you to receive full credit.

$$f(x) = \begin{cases} (x-1)^2, & x \leq -1; \\ x^2 - 1, & x > -1. \end{cases}$$

#4. (16 points)

(4a) (8 points) Let $f(x) = 3x + 7$. For any $\epsilon > 0$, find the largest value of δ such that if $|x + 3| < \delta$, then $|f(x) + 2| < \epsilon$. Express your answer in terms of ϵ .

(4b) (8 points) Given that $4x \leq f(x) \leq 2x^4 - 2x^2 + 4$ for all x , determine $\lim_{x \rightarrow 1} f(x)$. Your work should be presented neatly and logically. It must contain steps that mathematically justify your answer using theorems (Name any theorems that you are using) or properties covered in our Calculus course and must include enough detail to provide a logical and complete solution to the question asked in order for you to receive full credit.

#5. (16 points) Please justify all your work. Name any theorems that you might be using to solve the following two problems, explain any preconditions and why they are satisfied.

(5a) (8 points) Let: $f(x) = 2x^3 - 3x^2 + x - 2$. Show that equation $f(x) = 0$ has a solution on $(0, 2)$.

(5b) (8 points) Find all asymptotes (whether vertical or horizontal) of the following function: $f(x) = \frac{x^2 + 1}{x^2 - x}$.

#6. (18 points)

(6a) (12 points) Use the limit definition of the derivative to find $f'(x)$ for:

$$f(x) = 100x^2 + 99x + 97.$$

(6b) (6 points) Use the information from (a) to determine the slope of the line tangent to $f(x)$ at $x = 0$ and write the equation of the tangent line.

This page intentionally left blank