Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer _________________________ **Section (01, 02, 03, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 131 **Final Exam** **December 17, 2018**
1:00-3:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are seven (7) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do **not** use a calculator, reference materials, or paper other than a booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	14	
2	14	
3	14	
4	14	
5	14	
6	14	
7	14	
Free	2	2
TOTAL	100	

#1. Car A is 16 kilometers to the west of car B. At noon, Car A begins driving east at a constant speed 4 kilometers/hour, and car B begins driving north at a constant speed of 3 kilometers/hour.

(a) (5 points) Let x be the distance that car A has traveled since noon, and let y be the distance that car B has traveled since noon. Let z be the distance between the two cars. Find an equation that relates z to x and y.

(b) (2 points) Find the values of x and y after the cars have been driving for 2 hours.

(c) (7 points) Find the rate at which z is increasing after the cars have been driving for 2 hours.

#2. Suppose that $f(x)$ is a function whose first and second derivatives exist for all x. Suppose that $f(0) = 2$ and $f(1) = -3$. Clearly indicate any theorems you use and why they are relevant.

(a) (6 points) Show that there is a number a in the interval $(0, 1)$ such that $f'(a) = -5$.

(b) (8 points) Suppose that there is a number b in the interval $(1, 2)$ such that $f'(b) = 7$. Show that there is a number c between b and the number a from part (a) such that $f''(c) > 6$.

#3. Let $f(x) = \dfrac{x^2+1}{(x+1)^2}$. Then $f'(x) = \dfrac{2x-2}{(x+1)^3}$ and $f''(x) = \dfrac{8-4x}{(x+1)^4}$.

(a) (2 points) Find the domain of f.

(b) (2 points) Find the interval(s) on which f is increasing.

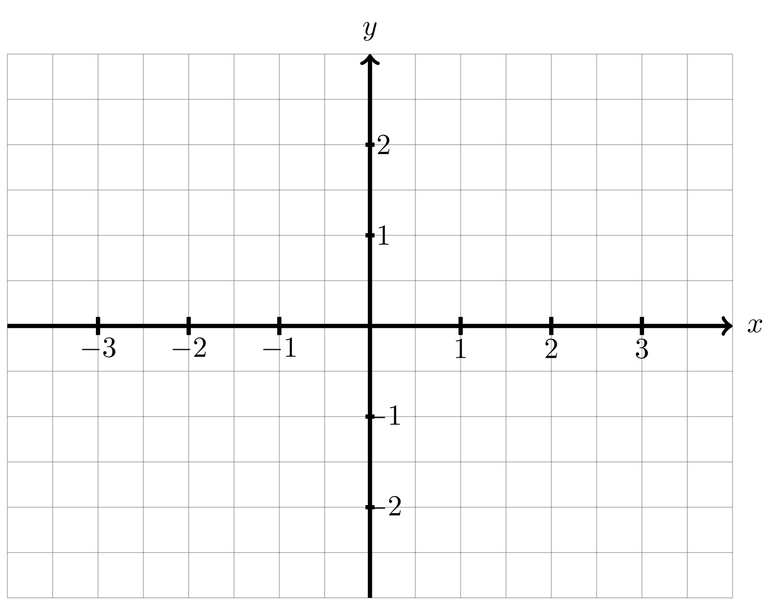
(c) (2 points) Find the interval(s) on which f is concave up.

(#3 continued)

(d) (2 points) Find the x-coordinates of the local maxima, local minima, and points of inflection of the graph of $f(x)$, if any.

(e) (3 points) Find the equations of the horizontal and vertical asymptotes of $f(x)$, if any.

(f) (3 points) Use the information obtained in parts (a)-(e) to sketch the graph of $y = f(x)$.



#4. Use L'Hospital's rule to evaluate the following limits.

(a) (3 points) $\lim_{x \to \infty} \frac{2x^2 + 1}{(x-3)^2}$

(b) (4 points) $\lim_{x \to 0^+} \frac{1}{x} - \frac{1}{\sin(x)}$

(c) (7 points) $\lim_{x \to 0^+} (e^x - x)^{\frac{1}{x}}$

#5. A rectangle has one of its sides on the x-axis and corners of the opposite side on the piece of the parabola $y = 12 - x^2$ above the x-axis.

(a) (6 points) Let $(x, 0)$ be the coordinates of the lower right vertex of the rectangle. Find an expression $A(x)$ for the area of the rectangle.

(b) (8 points) Find the largest possible area of such a rectangle.

#6. A particle is moving on the x-axis. Its acceleration as a function of time t is

$$a(t) = \cos(t) + t.$$

(a) (4 points) Find the most general antiderivative of $a(t)$.

(b) (4 points) Let $v(t)$ be the velocity of the particle at time t. Suppose that $v(0) = 0$. Find $v(t)$.

(c) (6 points) Let $s(t)$ be the position of the particle at time t. Suppose that $s(0) = 0$. Find $s(t)$.

#7. (This question is continued on the next page.) For this question, you may use the following identities without proof:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) (5 points) Consider the function $f(x) = x^2 + 5x - 2$ on the interval $[1, 4]$. Use three rectangles and right endpoints to approximate $\int_1^4 f(x)\, dx$.

(b) (3 points) Let $g(x)$ be an integrable function on the interval $[a, b]$. Write the *definition of the integral* $\int_a^b g(x)\, dx$, using right endpoints as sample points.

(#7 continued)
(c) (6 points) Consider the function $f(x) = x^2 + 5x - 2$ on the interval $[1, 4]$. Use the *definition of the integral* to evaluate $\int_1^4 f(x)\,dx$.

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